# CEPLAG – UMSS
**Centro de Planificación y Gestión**
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## IMPLEMENTATION OF MULTICRITERIA DECISION AIDING MODELS: Electre III, MacBeth, Promethee, Analytic Hierarchy Process and Naiade

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**Documentos de Trabajo**
**Número 7, abril de 2002**

Los "**Documentos de Trabajo**" del CEPLAG tienen el propósito de informar los avances de investigación que se están realizando en el CEPLAG. Así mismo, buscan contribuir al debate académico, tanto en el seno de la UMSS como en la región y en el país. Estos documentos son publicados gracias al apoyo del **Consejo Flamenco Interuniversitario de Bélgica (VLIR)** y de la **Universidad Católica de Leuven**.

**Cochabamba - Bolivia**

**Acknowledgments:**

This text was done thanks to the Belgian Government through the Vlaamse Interuniversitaire Raad (VL.I.R.). I wish to express my deep gratitude to the Belgian people whose contribution to the development cooperation is making it possible to accomplish these research activities in my country.

This work was also possible thanks to the valuable assistance of Mr. Jose Luis Murillo, whose expertise made it possible to carry out this case study in a comprehensive way.

# Content

# Implementation of Multicriteria Decision Aiding Models

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## Abstract

This document has been realized to exemplify the use of some Multiple Criteria Decision Aid methods, trough application to a hypothetical case. The chosen example is the analysis of a decision making process made by a person wishing to make the best selection of a second hand car in the market of Cochabamba, Bolivia. The reasons for using such an example is that the process of buying a car is easily understood by most people, avoiding polemic discussions about social, economical or environmental factors and focusing only on the process of decision making.


### *Structure of a Decision Making Process*

In a previous Working Paper [Sanchez, 2001] it has been said that any Decision Making Process can be decomposed into several phases, which we briefly list next:

- Phase Q: A general overview of the problem, trying to answer questions such as Who? What? When? Why? How? Where? …
- Phase I: Identification of stakeholders and the Decision Maker
- Phase A: Identification of alternatives
- Phase C: Criteria definition
- Phase D: Construction of a performance table using a proper set of performance measures, also called indicators
- Phase M: Application of a mathematical model
- Phase R: Recommendation of an action based on the results of the model

In this text we aim to deal with every phase of the Decision Making Process. There are two important remarks that are worth to be considered:

1) A Decision Making Process is not a linear process, therefore the different phases do not follow sequentially. The analysis of a decision making problem is a dynamic process in which all mentioned phases apply interactively and most of the time repeatedly.

2) A mathematical model is not a black box. The outcome of a mathematical model must be analyzed and interpreted as a *suggestion* before making a final decision. The involvement of intangible subjective criteria makes the decision process a scientific art, because it constitutes the development of imaginative skills as representation of the natural world by means of science.

## ***Phase Q: General overview***

We first start by introducing the protagonist of our story. Maria is a young Bolivian lady who wishes to buy a car. From time to time she practices outdoor activities such as fishing and trekking and likes to make frequent trips to the country-side to take advantage of wonderful opportunities of nature sightseeing offered by this part of the world, Bolivia. For reasons that we are not going to discuss here, she decided to buy a four-wheel drive four-doors pick-up. She approached her decision concerning the type of vehicle she was going to buy via the use of a Multi-Criteria Decision Aiding method bearing in mind her preferences about the type of use she was going to make of the car, the reliability of the branch, the availability of spare parts and others. Her problems started again when she began looking for the desired pick-up on the second hand car market of Cochabamba. Many different possible choices, different prices and several sellers. She was confused about the way of choosing the best pick -up. To make a reasonable decision, several aspects had to be considered in the analysis, most of them of subjective nature. She could not trust sellers' opinion regarding their own cars, so expert advise had to be found. There was much uncertainty concerning the real condition of the offered pick-ups and she did not have enough knowledge nor the willingness to evaluate by herself some important aspects, specially those concerning mechanical conditions. Therefore she decided to use once Multi-Criteria Decision Aiding methods to be sure of making the best possible choice. After all, this was the investment of every part of her funds and consequently it deserved great attention.

After a pen-and-paper analysis of the situation, she found that there were several aspects that had to be considered in the choice of a second hand car:

- The first aspect she thought to use as criterion was the model of the vehicle. This would give her an idea of the age of the vehicle and consequently the amount of use given of the vehicle by the previous owner. But at this point some questions arose: What if the car is old but very well maintained and not intensively used? Or, conversely, what if the car is of a recent model but was driven intensively under hard conditions? Additional information about the quantity of kilometers traveled and the type of use given to the car would give her a better insight in to the story. The model of the vehicle could not be used as the only indicator of car condition.

- The price of the car should not fall beyond her budget. The asked price had to be set according to the current general state of the vehicle. Although the price is a very important aspect, it is not the only economic aspect to consider. The amount of taxes (in Bolivia, the newer the vehicle, the higher its tax fees), fuel consumption levels (normally, newer vehicles consume less fuel), the price of spare parts (newer vehicles have more complex engines and more expensive spare-parts) and depreciation (10% of the price each year) have to be accounted as part of the price as well.

- Mechanical aspects such as the engine condition (clutch, transmission, crown, breaks, direction, suspension, electric system, wheels and others) had to be checked before buying the car. Maria, although a self-sufficient lady, was terrified about the need of checking those mechanical aspects. She would need help. Two kind of mechanical problems could arise. Mechanical problems that need to be solved immediately and thus directly affect the price, and those problems related to normal deterioration through the use of the vehicle, which are related to the quantity of kilometers traveled and the type of use given to the car. The latter aspect can not be solved and consequently does not represent a cost, but of course the price is affected by it.

- The car should look nice. Chassis condition (outside metal), painting, color, and a good general look are aspects that, although perhaps not the most important ones, could make a difference and therefore deserve being accounted.

- The car should be comfortable enough to make long trips. Ergonomic aspects such as seats comfortability, steering-wheel position, the availability of optional accessories such as stereo, air conditioning, and the condition of inside cabin such as the seat cover, roof cover, command panel, etc., are all aspects that make a trip comfortable and the daily use of the car pleasing . Most of these aspects can be transformed into the cost of buying them, but Maria knows that this would never happen because once the car is bought, she will have no extra money nor the motivation to make such changes.

- Given the kind of use Maria was going to give to the car, safety aspects were also important. She wanted to verify the seat belt availability, break and direction condition and related details of the car.

- The vehicle should be free of legal problems. As she was looking for a second hand car, there was some risk of swindle from previous owners. Bolivia has gone trough periods of high corruption in which, in the past, it was easy to “pay” for false documents of vehicles. There are regions well known for this. This makes it risky to own a car whose documents were made in such regions, but this risk is difficult to measure because, of course, there are not much data available concerning those regions and their propensity to misbehavior.

- Maria is aware of pollution problems and she does not want to contribute to polluting the environment. She would like to drive a car in which recyclable and recycled materials were used in its manufacturing. Also the kind of fuel used by the car was an important aspect. Natural gas is a very clean and cheap fuel but less powerful than gasoline. It could affect speed and engine power for mountain crossing. As there are not original engines fueled by natural gas, those vehicles using natural gas are mechanically transformed into natural gas. It is proven that this transformation is feasible and safe enough, but there are rumors that it shortens the life cycle of the engine. Diesel pollutes a lot but it is a powerful fuel, cheaper than gasoline, but more expensive than natural gas. These were all important aspects, but she was now confronted with a situation in which she may pay extra to have a non-polluting car.

- Maria wanted to avoid her car from being stolen. While driving a new and nice-looking car may be safe in terms of risk of accident, it could be dangerous in terms of risk of theft or, even worse, assault. The availability of alarm, lock and insurance are positive factors, but it would affect the price.

Maria realized the difficulty of the choice. Many of the aspects to consider were opposing to each other and almost all of them were of subjective or uncertain nature.

## Phase C: Definition of main criteria

After a process of summarization and analysis, she decided to use the following summarized set of criteria:

- ⇨ Economic aspects, including complete repair
- ⇨ Technical condition after complete repairing
- ⇨ Aesthetics
- ⇨ Comfort
- ⇨ Environmental aspects
- ⇨ Risk of theft

## Phase A: Identification of alternatives

At a given point in time Maria found several possible alternatives on the second hand market of Cochabamba. After an analysis, several of them were neglected because they were too expensive, too old, too bad looking or of doubtful origin. The neglected alternatives surpassed, inside the Decision Maker’s mind, some tangible or subjective threshold.

Three alternatives remained:

| **The black** | **The green** | **The grey** |
|---|---|---|
|  |  |  |

## Phase D: Construction of a General Performance Table

Now, what she has to do is to find performance measures that describe the actual condition of every important aspect she wants to consider in her decision, a kind of descriptive picture that helps her to remember and consider important characteristics of the vehicles.

Facing the technical aspects was a task for expert advice. Expert technicians made use of a table in which they expressed their perceptions about the car condition, as shown next:

**Technical diagnosis made by experts in the field of mechanics**:

| ELEMENT | SCALE |
|---|---|
| ⇨ **Engine** | |
| Distribution mechanism (chain or strap) : | Very good/Good/Medium/Bad/Very bad |
| Oil condition: | Very good/Good/Medium/Bad/Very bad |
| Spark plugs: | Very good/Good/Medium/Bad/Very bad |
| Oil leaks: | Yes/No |
| ⇨ **Clutch** | |
| Disk or press | Very good/Good/Medium/Bad/Very bad |
| Liquid leaks | Yes/No |
| ⇨ **Transmission (gear shift)** | |
| Oil condition: | Very good/Good/Medium/Bad/Very bad |
| Synchronizers: | Very good/Good/Medium/Bad/Very bad |
| Oil leaks: | Yes/No |
| Noises: | Very High/High/Medium/Low/Very Low |
| Cardan joints: | Very good/Good/Medium/Bad/Very bad |
| ⇨ **Differential (crown)** | |
| Oil condition: | Very good/Good/Medium/Bad/Very bad |
| Oil leaks: | Yes/No |
| Noises: | Very High/High/Medium/Low/Very Low |
| ⇨ **Breaks** | |
| Friction pads | Very good/Good/Medium/Bad/Very bad |
| Discs: | Very good/Good/Medium/Bad/Very bad |
| Break drums: | Very good/Good/Medium/Bad/Very bad |
| Liquid leaks: | Yes/No |
| Hand break: | Very good/Good/Medium/Bad/Very bad |
| ⇨ **Direction** | |
| Steering knuckles: | Very good/Good/Medium/Bad/Very bad |
| Arms: | Very good/Good/Medium/Bad/Very bad |
| Liquid leaks: | Yes/No |
| ⇨ **Suspension** | |
| Shock absorbers: | Very good/Good/Medium/Bad/Very bad |
| Swivels: | Very good/Good/Medium/Bad/Very bad |
| Spring bushings: | Very good/Good/Medium/Bad/Very bad |
| Bar bushings: | Very good/Good/Medium/Bad/Very bad |
| ⇨ **Electric system** | |
| Condition of lights and electric appliances | Very good/Good/Medium/Bad/Very bad |
| ⇨ **Wheels** | |
| Tires condition | Very good/Good/Medium/Bad/Very bad |
| Bearings set | Very good/Good/Medium/Bad/Very bad |
| Alignment | Very good/Good/Medium/Bad/Very bad |
| Balance | Very good/Good/Medium/Bad/Very bad |
| ⇨ **Coachwork** | |
| Body work and painting | Very good/Good/Medium/Bad/Very bad |
| Chassis | Very good/Good/Medium/Bad/Very bad |
| Noises | Very High/High/Medium/Low/Very low |
| **Total cost of repair**: | [$]: |
| **General state of the vehicle:** | Very good/Good/Acceptable/Bad/Very Bad |

The outcome of such a diagnosis is the amount of money needed to repair the elements in poor order and a subjective evaluation of the overall technical performance.

With this data, the following General Performance Table can be built. This table is useful to record relevant characteristics at the moment of inspection of alternatives.

GENERAL PERFORMANCE TABLE

| Id | Aspect | Meas. Unit | Alternative | | |
|---|---|---|---|---|---|
| | | | **Black** | **Green** | **Grey** |
| **G** | **GENERAL CHARACTERISTICS** | | | | |
| G1 | Model | Year | 1,991 | 1,997 | 1,997 |
| G2 | Distance covered | Km | 95,144 | 50,480 | 26,286 |
| G3 | Type of use | Scale | Delicate | Moderate | Hard |
| | Hard / Moderate / Delicate | | | | |
| **P** | **ECONOMIC ASPECTS** | | | | |
| P1 | Asked price | $ | 10,500 | 13,500 | 16,000 |
| P2 | Amount of taxes paid this year | $ | 100 | 150 | 150 |
| P3 | Fuel consumption | Km/lt | 8 | 10 | 10 |
| P4 | Cost of repair to the best possible condition | $ | 690 | 260 | 85 |
| P5 | Depreciation per year (10% of asked price) | $ | 1,050 | 1,350 | 1,600 |
| P6 | Total price (P1+P2+P4+P5) | $ | **12,340** | **15,260** | **17,835** |
| P7 | Price of spare-parts in the market (difference with Toyota) | % | Normal | High | High |
| | Very High / High / Normal / Cheap / Very Cheap | | | | |
| **T** | **TECHNICAL CONDITION** | | | | |
| T1 | Technical condition after complete repair | Scale | Acceptable | Good | Good |
| | Very Good / Good / Acceptable / Needs Repair / Serious Problems | | | | |
| **A** | **AESTHETICS** | | | | |
| A1 | Body (outside metal) | Scale | Some | None | None |
| | Serious crash at: Front / Back / Lateral \|\| Small crashes: A Lot / Some / Little / None | | | | |
| A2 | Painting | Scale | Part Cracked | Oven dried | Opaque |
| | Sun burned / Opaque / Partially Cracked / Oven Dried / Brilliant / … | | Oven dried | | |
| A3 | Color | Scale | I Like | Indifferent | I Dislike |
| | I Like Much / I Like / Indifferent / I Don't Like / I Dislike | | | | |
| A4 | Noise while driving | Scale | Some Noise | Little Noise | Almost Nothing |
| | A Lot of Noise / Some Noise / Little Noise / Almost Nothing / Nothing | | | | |
| A5 | General look | Scale | I Like Much | Indifferent | I Don't Like |
| | I Like Much / I Like / Indifferent / I Don't Like / I Dislike | | | | |
| **C** | **COMFORT** | | | | |
| C1 | Ergonomics (seats, steering wheel position, …) | Scale | Normal | Normal | Normal |
| | Very Comfortable / Normal / Uncomfortable | | | | |
| C2 | *Optional accessories* | | | | |
| C3 | Stereo | yes/no | Yes | Yes | Yes |
| C4 | Air conditioning | yes/no | No | No | Yes |
| C5 | Other: | Scale | No Device | No Device | No Device |
| | Highly Appreciated Device / Indifferent / I Dislike the Device / No Device | | | | |
| C6 | *Condition of inside cabin* | | | | |
| C7 | Seat cover | Scale | Good | Good | Good |
| | Very Good / Good / Acceptable / Needs Repair / Serious Problems | | | | |
| C8 | Roof cover | Scale | Good | Good | Good |
| | Very Good / Good / Acceptable / Needs Repair / Serious Problems | | | | |
| C9 | Command panel | Scale | Acceptable | Good | Good |
| | Very Good / Good / Acceptable / Needs Repair / Serious Problems | | | | |
| **L** | **LEGAL ASPECTS** | | | | |

| | | | | | |
|---|---|---|---|---|---|
| L1 | Risk of swindle from previous owner | Scale | Low | Very Low | Very Low |
| | Very High / High / Medium / Low / Very Low | | | | |
| L2 | Complete fulfillment of legal obligations | yes/no | Yes | Yes | Yes |
| **E** | **ENVIRONMENTAL ASPECTS** | | | | |
| E1 | Fuel type | Type | Gas/Gasoline | Gasoline | Gasoline |
| E2 | Recyclable materials | yes/no | no | no | no |
| **R** | **RISK OF -THEFT** | | | | |
| R1 | Is it too attractive to thieves? | Scale | Very attract | Very attract | Indifferent |
| | Very attractive / Attractive/ Indifferent / Low probability / No risk | | | | |
| R2 | Alarm | yes/no | yes | no | yes |
| R3 | Lock | yes/no | yes | no | no |

## ***Phase M: Implementing Decision Aiding Models***

In what follows, a step by step implementation of five Decision Aiding Models is presented.

## IMPLEMENTING ELECTRE III

Electre III , developed by Bernard Roy in 1968, was built based on the *outranking relation* for modeling the Decision Maker's preferences. An outranking relation is a *binary relation* S such that, given two alternatives *a* and *b*, "*a* outranks *b*" if there are enough arguments to decide that *a* is at least as good as *b*, while there is no essential reason to refute that statement [Vincke, 1992].

### Step 1: Editing project reference

Specification of the owner and a short description of the problem.

⇨*Owner*: Maria.
⇨*Description*: The choice of the best four-wheel-drive four-doors vehicle that Maria can get from the second hand car market of Cochabamba.

### Step 2: Defining criteria

Due to software constraints of the academic version, only 5 criteria are allowed in the analysis. For this reason, the criterion "Risk of theft" has been skipped.

In order to use the software, Maria must enter descriptive data associated to the criteria defined during phase C.

- ☑ *The code of every criterion*
- ☑ *The weight of every criterion*
- ☑ *Direction of preferences of each criterion (Increasing/Decreasing)*

The *weight* is a value expressing the relative importance of one criterion with respect to other criteria. Electre normalizes the values, so that all of them amount to 1. Maria's inputs are shown next:

| Criterion | Code | Weight | Direction of preference |
|---|---|---|---|
| Economic aspects | ECON | 29.5 | Decreasing |
| Engine condition | ENGI | 31.1 | Increasing |
| Aesthetics | AEST | 03.0 | Increasing |
| Comfort | COMF | 10.0 | Increasing |
| Environmental aspects | ENVI | 26.3 | Increasing |

## Step 3: Defining alternatives

The set of alternatives was identified during Phase A.

- ⇨ The Black
- ⇨ The Green
- ⇨ The Grey

## Step 4: Evaluating alternatives: the Performance Table

To run this software, after defining criteria and alternatives Maria has to enter data concerning alternative *performances* upon criteria. A *performance table* is required, in which alternatives are evaluated one by one, respect to every criterion. Maria has to imagine *scales of measurement* that describe the particular state of every alternative on each criterion. To do so, Maria can use the data available on the General Performance Table built during Phase D, which is useful to see and remember characteristics of every alternative.

Maria's inputs are presented as follows:

| Criterion | Scale of measurement |
|---|---|
| Economic aspects | From 0 to infinite |
| Engine condition | *New=9; Very Good=8; Good=7; More or Less Good=6; Acceptable=5; Somewhat Bad=4; Bad=3; Very Bad=2; Awful=1* |
| Aesthetics | *Excellent=9; Very Good=8; Good=7; More or Less Good=6; Indifferent=5; Somewhat Bad=4; Bad=3; Very Bad=2; Awful=1* |
| Comfort | *Excellent=9; Very Good=8; Good=7; More or Less Good=6; Indifferent=5; Somewhat Bad=4; Bad=3; Very Bad=2; Awful=1* |
| Environmental aspects | *If fueled by natural gas=3; If fueled by gasoline=2; If fueled by diesel=1* |

For example, she defined that the aesthetics of the black vehicle was "*very good*" (i.e. 8), while it was "*more or less good*" (i.e. 6) for the green vehicle. The result of the evaluation process is the performance table shown next:

ELECTRE PERFORMANCE TABLE

| | Economic | Engine cond. | Aesthetics | Comfort | Environmental |
|---|---|---|---|---|---|
| **Black** | 12.340 | Somewhat bad | Very Good | More or less Good | Natural gas |
| **Green** | 15.260 | Good | More or Less Good | Good | Gasoline |
| **Grey** | 17.835 | Very Good | Indifferent | Good | Gasoline |

Or its equivalent:

ELECTRE PERFORMANCE TABLE

| | Economic | Engine cond. | Aesthetics | Comfort | Environmental |
|---|---|---|---|---|---|
| **Black** | 12.340 | 5 | 8 | 6 | 3 |
| **Green** | 15.260 | 7 | 6 | 7 | 2 |
| **Grey** | 17.835 | 8 | 5 | 7 | 2 |

## Step 5: Defining thresholds

A threshold is a limit that is chosen to establish the point from which an element changes its class. Thresholds are used to take into account hesitation. Within Electre methods three types of thresholds are used:

- The *Preference Threshold*, denoted by *p*, defines the point from which an element is strictly preferred in relation to another element. For example, a vehicle which price is $10.000 is strictly preferred to a vehicle which price is $11.000.

- The *Indifference Threshold*, denoted by *q*, defines an interval within which two elements are considered equal. For example, a price of $10.000 can be considered, for comparison purposes, equivalent to a price of $10.100.

- The *Veto Threshold*, denoted by *v*, is a limit beyond which the credibility of the outranking relation of two alternatives is refused. For example, if the difference of price of two vehicles is bigger than $1.000, then the credibility of that outranking relation is refused.

Thresholds must be set by means of the following formula:

$$T_j=\alpha * g_j(a)+ \beta,$$

where:

- $T_j$ stands for *Indifference, Preference or Veto thresholds*, on the $j^{th}$ criterion .
- $g_j(a)$ stands for alternative *a*'s performance on the $j^{th}$ criterion.
- $\alpha$ is a coefficient from 0 to 1. It can be seen as a percentage of the alternative's performance that the Decision Maker is willing to tolerate.

- $\beta$ is a coefficient that can be interpreted as the amount that the Decision Maker is willing to tolerate in addition to the percentage. It is expressed in the same units of the performance scale.

In order to establish thresholds, Maria has to set values of parameters $\alpha$ and $\beta$ according to her perceptions about the limits of preference and indifference.
In the following table we show Maria's coefficients for threshold setting. Maria has not indicated any veto thresholds.:

THRESHOLDS TABLE:

| | COEFFICIENT | ECON | ENGI | AEST | COMF | ENVI |
|---|---|---|---|---|---|---|
| **Preference threshold** | $\alpha$ | 0 | 0 | 0 | 0 | 0 |
| | $\beta$ | 1000 | 2 | 3 | 2 | 0 |
| **Indifference threshold** | $\alpha$ | 0 | 0 | 0 | 0 | 0 |
| | $\beta$ | 500 | 1 | 2 | 1 | 0 |
| **Veto threshold** | $\alpha$ | 0 | 0 | 0 | 0 | 0 |
| | $\beta$ | 0 | 0 | 0 | 0 | 0 |

## Step 6: Computing

This text is based on Vincke, 1992.

Being: $w_j$ the weight associated to each criterion $g_j$
$g_j(a_i)$ alternative $a_i$'s performance with respect to criterion $g_j$
$q_j$ the indifference threshold (of type $T_j$)
$p_j$ the preference threshold, (of type $T_j$)

the following *concordance index* c(*a*,*b*) is calculated for each ordered pair of actions (*a*,*b*):

$$c(a,b)=(1/W)\sum_{j=1}^{n} w_j c_j(a,b),$$

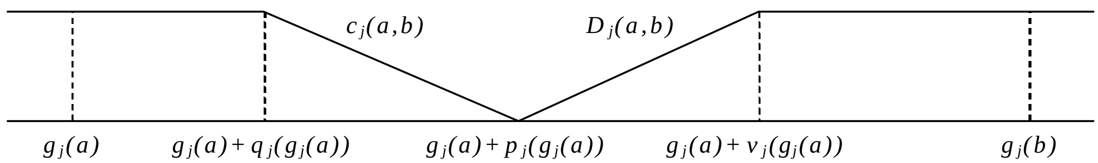
where: $W=\sum_{j=1}^{n} w_j$

$$c_j(a,b) = \begin{cases} 1 \text{ if } g_j(a)+q_j(g_j(a)) \geq g_j(b) \\ 0 \text{ if } g_j(a)+p_j(g_j(a)) \leq g_j(b) \\ \text{linear between the two.} \end{cases}$$

Then, the *discordance index* is calculated as follows:

$$D_j(a,b) = \begin{cases} 0 \text{ if } g_j(b) \leq g_j(a)+p_j(g_j(a)) \\ 1 \text{ if } g_j(b) \geq g_j(a)+v_j(g_j(a)) \\ \text{linear between the two.} \end{cases}$$

being $v_j$ the veto threshold, meaning that any credibility for the outranking of *b* by *a* (i.e.*a*S*b*) is refused if $g_j(a) \geq g_j(b) + v_j(g_j(b))$.




Next, the *degree of outranking* of every pair of alternatives ($a,b$) is calculated as follows:

$$S(a,b)= \begin{cases} c(a,b) \text{ if } D_j(a,b) \leq c(a,b), \forall j \\ c(a,b) * \Pi_{j\in J(a,b)} (1-D_j(a,b))/(1-c(a,b)), \text{ otherwise.} \end{cases}$$

where $J(a,b)$ is the set of criteria for which $D_j(a,b) > c(a,b)$.

The ranking of alternatives is performed by *distillation procedures*. To do so, a value $\lambda$=max$S(a,b)$ is determined, considering every pair of alternatives ($a,b$). Then, only the pairs of alternatives having arcs “sufficiently close” to $\lambda$ are considered through the use of the threshold $s(\lambda)$. Then, the *qualification* $Q(a)$ can be computed, which is the number of actions which are outranked by $a$, minus the number of actions which outrank $a$. The set of actions with the largest $Q(a)$ will be called *first distillate* $D_1$. If $D_1$ only contains one action, the distillation procedure starts again in $A \backslash D_1$; otherwise, the same procedure is applied inside $D_1$. If a distillate $D_2$ which is thereby obtained is a singleton, the procedure is started again in $D_1 \backslash D_2$ (except if the latter is empty); otherwise it is applied inside $D_2$, and so forth until $D_1$ is used-up entirely, before starting with $A \backslash D_1$. This procedure, called a *descending distillation chain*, yields a first complete preorder. A second complete preorder is obtained by an *ascending distillation chain*, in which the actions having the smallest qualification are first retained. These two preorders then are compared each other. If they are close, the Decision Maker is offered a “median preorder”. If they are very different, a more through study is suggested.

## Step 7: Getting results from Electre III

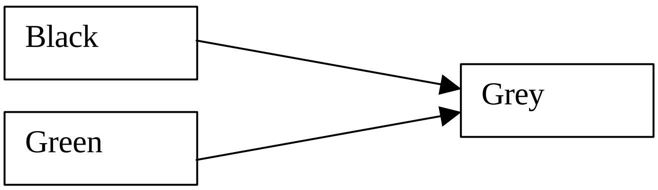
After calculation, Electre presents the following results.

Descending distillation:

Ascending distillation:


Ranks in final preorder: 1 for Green and Black
2 for Grey

Final graph:




### Comments on Electre III

The application of ELECTRE includes choosing indifference and preference thresholds for the criteria and eliciting the relative importance of criteria through establishment of weights. Setting weights of relative importance for each criterion is an important, but difficult task. Relative importance of criteria may not be clear for the Decision Maker and it may be difficult to extract a numerical value for a criterion. If this task is made without the assistance of a systematic preference model, the output can be misleading and values of relative importance may be arbitrary. Moreover, ELECTRE does not take into account uncertainty and hesitation of the Decision Maker at the time of establishing weights of relative importance of criteria. On the other hand, thresholds allow accountability of some types of uncertainty. Imprecision and variation with time may provoke small differences that can not be regarded as convincing evidence of preference. Nevertheless, definition of thresholds attain some sort of arbitrariness as well, because they constitute rigid limits normally established based on subjective perceptions. An additional source of arbitrariness presented in ELECTRE is the establishment of numerical values after measurement scales constructed by the Decision Maker in order to account subjective criteria.

## IMPLEMENTING MACBETH

MacBeth (Measuring Attractiveness by a Categorical Based Evaluation Technique) was developed by Carlos Bana e Costa and Jean-Claude Vansnick at University of Mons-Hainaut. MacBeth operates by building cardinal numerical scales for measuring attractiveness.

Three phases can be identified when applying MacBeth :

- The structuring phase:
  - Identification of critical success factors, also called Fundamental Points of View (FPV), which constitute the criteria to be used in the Decision Making Process.
  - Construction of descriptors for every FPV.
- The evaluation phase: Weighting of FPV’s
- Direction of action based on results from the Decision Making Process.

*Structuring phase*

## Step 1: Defining the FPV's

The basis for the structuring process of our example was done during what we called Phases Q, C, I, A and D. Criteria found during those phases constitute the Fundamental Points of View utilized in this model.

## Step 2: Building the descriptors table

To be able to build a Descriptors Table the Decision Maker has to imagine, for every FPV, "*impacts*" of different performance levels. In other words, Maria has to imagine a set of "states" of the vehicle regarding each FPV, and describe what would happen if the vehicle is on that "state".

There can be different types of descriptors:

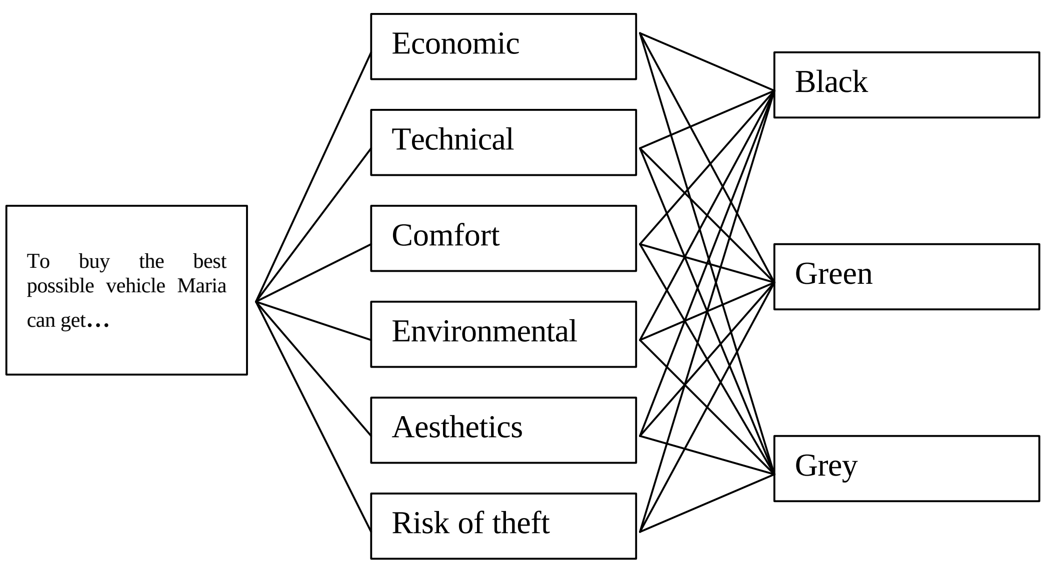
- ➡ Direct or natural: Descriptor levels directly reflect effects
- ➡ Indirect: Descriptor levels are indicators of causes
- ➡ Constructed: Descriptor levels are a definite set of reference levels
- ➡ Qualitative or quantitative (or even pictorial)
- ➡ Continuous or discrete

In the following table we show descriptors of every FPV of our example. Due to the nature of criteria, all descriptors belong to the type "Constructed":

| DESCRIPTORS TABLE<br>(A DESCRIPTOR IS AN ORDERED SET OF IMPACT LEVELS) | | |
|---|---|---|
| **FPV** | **Impact description** | **Impact Level** |
| **Economic aspect** | • The price is below my budget.<br>• The price is higher than my budget, but I can still finance the difference.<br>• The price is much higher than my budget, I can hardly afford it. | A$ [Good]<br>B$ [Neutral]<br>C$ |
| **Technical condition after complete repairing** | • The vehicle will run for another 40.000 Km without problems.<br>• The vehicle will run for another 40.000 Km with minor repairs.<br>• The vehicle will need some repair every 10.000 Km.<br>• The vehicle will need a serious repair soon.<br>• The vehicle will cause serious and continuous mechanical problems. | AT<br>BT [Good]<br>CT [Neutral]<br>DT<br>ET |
| **Aesthetics** | • The vehicle is admired by most people.<br>• The vehicle is admired by those people who like this kind of vehicle.<br>• The vehicle is just another vehicle in the parking yard.<br>• The vehicle provokes critics about its beauty.<br>• The vehicle inspires shame. | AA<br>BA [Good]<br>CA [Neutral]<br>DA<br>EA |

| | | |
|---|---|---|
| **Comfort** | • It is possible to make a very comfortable 8 Hrs. trip in hard climate across rough surfaces.<br>• It is quite tiring to make an 8 Hrs. trip in hard climate across rough surfaces, but still possible.<br>• I would not take children for an 8 Hrs. trip in hard climate across rough surfaces.<br>• Only when necessary I would make an 8 Hrs. trip in hard climate across rough surfaces.<br>• Only when urgent I would make an 8 Hrs. trip in hard climate across rough surfaces. | AC [Good]<br>BC [Neutral]<br>CC<br>DC<br>EC |
| **Environ-mental aspects** | • The vehicle pollutes almost nothing.<br>• The vehicle pollutes within "normal" or "socially accepted" limits.<br>• The vehicle pollutes more than what is desirable. | AE [Good]<br>BE [Neutral]<br>CE |
| **Risk of theft** | • I can leave the vehicle outside without alarm nor lock.<br>• I can leave the vehicle outside, but it would be wise to have alarm or lock.<br>• I can leave the vehicle outside, but I must put alarm and lock.<br>• I can leave the vehicle outside, but I must put alarm, lock and a person watching over. | AR<br>BR [Good]<br>CR<br>DR [Neutral] |

At this point, a graph showing FPV's and alternatives can be drawn as follows:




***Evaluation phase***

## Step 3: Building cardinal value functions

The goal of the evaluation phase is to measure *relative attractiveness* of alternatives concerning every FPV. To do so, it is necessary to build *cardinal value functions* upon relative descriptors and to assign *scaling constants* to the apparently reasonable (i.e. plausible) ranges of impact levels. This is done with the help of MacBeth software.

In order to perform comparisons, the following *comparison scale* must be used:

MACBETH COMPARISON SCALE:

| CODE | VERBAL STATEMENT | VALUE |
|---|---|---|
| $C_0$ | no difference of attractiveness | 0 |
| $C_1$ | very weak difference of attractiveness | 1 |
| $C_2$ | weak difference of attractiveness | 2 |
| $C_3$ | moderate difference of attractiveness | 3 |
| $C_4$ | strong difference of attractiveness | 4 |
| $C_5$ | very strong difference of attractiveness | 5 |
| $C_6$ | extreme difference of attractiveness | 6 |

The process of comparison is performed as follows. Taking as example the "Aesthetics" FPV (see Descriptors Table):

⇨ Question: How is the <u>*difference of attractiveness*</u> between “to buy a vehicle that is admired by most people”, and “to have a vehicle that is admired only by those who like this kind of vehicles”? (i.e. AA Vs. BA)

⇨ Answer: There is a *weak difference of attractiveness* ($C_2$) between the two statements.

⇨ Question: How much more attractive would be “to buy a vehicle that is admired by most people”, than “to buy a vehicle that would be just another vehicle in the parking yard”? (i.e. AA Vs. CA)

⇨ Answer: It would be *moderately more attractive* ($C_3$) to buy a vehicle that is admired by most people, than to buy a vehicle that would be just another vehicle in the parking yard.

⇨ And so forth…

One important characteristic of MacBeth is that information of preferences is obtained indirectly. Relative preference of two alternatives is obtained throughout the *difference of attractiveness* between them. For instance, the *difference of attractiveness* between a *good* and a *very good* alternative, can be the same as the *difference of attractiveness* between a *bad* and a *very bad* alternative. Numerical values corresponding to verbal statements for impact levels of a criterion (in this case “Aesthetics”), are placed in a comparison matrix like this:

MACBETH MATRIX of COMPARISONS:

| AESTHETICS | AA | BA | CA | DA | EA |
|---|---|---|---|---|---|
| **AA** | 0 | 2 | 3 | 4 | 6 |
| **BA** | | 0 | 2 | 3 | 5 |
| **CA** | | | 0 | 3 | 5 |
| **DA** | | | | 0 | 3 |
| **EA** | | | | | 0 |

Step 4: Computing

As said before, MacBeth produces a ranking of alternatives based on indirect information concerning the difference of attractiveness between two pairs of stimuli. Based on the matrix of categorical judgments, MacBeth verifies if there exists a numerical scale $\varphi$ on S that satisfies the two following conditions:

Condition 1 (ordinal condition):

$\forall\, x,y \in$ S: $\varphi(x) > \varphi(y) \Leftrightarrow x$ is more attractive than $y$

Condition 2 (semantic condition)

$\forall$ k,k'$\in${1,2,3,4,5,6}, $\forall\, x,y,w,z \in$ S with $(x,y) \in C_k$ and $(w,z) \in C_{k'}$ : $k \geq k'+1 \Rightarrow \varphi(x)-\varphi(y) > \varphi(w)-\varphi(z)$

If conditions 1 and 2 can be satisfied, the model determines, from all possible scales, a particular scale $\mu$ on S by a procedure that consists essentially of solving the following linear program.

Set of stimuli: S={$s_1,s_2,...,s_n$)

Relation P on S, such that:

$s_1$ is at least as attractive as $s_2,...,s_{n-1}$, and is more attractive than $s_n$

$s_2$ is at least as attractive as $s_3,...,s_n$

$s_3$ is at least as attractive as $s_4,...,s_n$

...

$s_{n-1}$ is at least as attractive as $s_n$

Positive variables: $\varphi(s_i)$ , $i \in \{1,2,...,n\}$

Objective function: min $\varphi(s_1)$

Constraints:

(1) $\varphi(s_n) = 0$

(2) $\forall\, x,y \in$ S, such that $(x,y) \in C_0$: $\varphi(x)=\varphi(y)$

(3) $\forall$ k,k'$\in${1,2,3,4,5,6} such that k>k', $\forall (x,y) \in C_k$ y $\forall (w,z) \in C_{k'}$:

$\varphi(x)-\varphi(y) \geq \varphi(w)-\varphi(z) + k - k'$

## Step 5: Getting descriptors' preference ranking from MacBeth

After running MacBeth six times, one for each FPV, Maria can summarize the results in the following table:

MACBETH TABLE FOR RESULTS OF THE COMPARISON PROCESS:

| **ECONOMIC** | | **TECHNICAL** | | **AESTHETICS** | | **COMFORT** | | **ENVIRONMENTAL** | | **RISK OF THEFT** | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A$ | 11=64.7% | AT | 21=34.4% | AA | 12=34.3% | AC | 13=41.9% | AE | 9=64.3% | AR | 11=47.8% |
| B$ | 6=35.3% | BT | 18=29.5% | BA | 10=28.6% | BC | 9=29% | BE | 5=35.7% | BR | 8=34.8% |
| C$ | 0 | CT | 13=21.3% | CA | 8=22.9% | CC | 6=19.4% | CE | 0 | CR | 4=17.4% |
| | | DT | 9=14.8% | DA | 5=14.3% | DC | 3=9.7% | | | DR | 0 |
| | | ET | 0 | EA | 0 | EC | 0 | | | | |

## Step 6: Setting weights of preference through fictitious alternatives

The next step in the MacBeth Decision Making Process is to determine, again based on indirect information, weights of preference on the criteria. To do so, Maria has to imagine seven fictitious vehicles. The first one, [*a0*] , is a vehicle with the worst plausible impact on all criteria. The second vehicle has the best possible impact on the “Economic” criterion, but the worst plausible impact on the remaining criteria. The third vehicle has the best possible impact on the “Technical” criterion, but the worst plausible impact on the remaining criteria, and so forth.

Once Maria has imagined seven fictitious alternatives (i.e. six corresponding to FPV’s plus one for null), she has to use MacBeth software again to compare overall attractiveness of these seven fictitious alternatives.

The academic version of MacBeth software allows a maximum of 5 stimuli on its judgment matrices. For this reason two criteria have to be skipped from the analysis. Therefore, the set of criteria to be used from now on is: *Economic*[*a$*], *Technical*[*aT*], *Comfort*[*aC*] and *Environmental*[*aE*]. Due to operational reasons, *Risk of theft* and *Aesthetics* are taken out from the analysis. These two criteria were still considered in previous steps of the process with the purpose of enriching this text with the examples of their impact descriptors. The reader should not be confused and just skip those criteria from now on.

The judgment matrix of fictitious alternatives and MacBeth’s results are shown next:

MACBETH JUDMENT MATRIX AND REULTS:

| | **a$** | **aE** | **aT** | **aC** | **a0** | **Scores** |
|---|---|---|---|---|---|---|
| **a$** | 0 | 1 | 2 | 5 | 6 | 31.82% |
| **aE** | | 0 | 2 | 5 | 6 | 29.55% |
| **aT** | | | 0 | 4 | 6 | 25.00% |
| **aC** | | | | 0 | 5 | 13.64% |
| **a0** | | | | | 0 | 0.00% |

There are two tasks remaining:

- ⇨ to evaluate alternatives’ performances with respect to each criterion.
- ⇨ to aggregate results.

## Step 7: Evaluating alternatives

Evaluation of alternatives with respect to each criterion can be done using the Impact Descriptors (see Descriptors Table – Step 2). For example, Maria defined that, with respect to “Comfort”, it can be *quite tiring to make an 8 hours trip* (BC) with alternatives Black and Green, while *it is possible to make a comfortable 8 hours trip* (AC) with alternative Grey.

Results of the evaluation process are shown next:

MACBETH DESCRIPTORS TABLE:

| | **Economic** | **Technical** | **Comfort** | **Environmental** |
|---|---|---|---|---|
| **Black** | A$ | CT | BC | AE |
| **Green** | B$ | BT | BC | BE |
| **Grey** | C$ | AT | AC | BE |

During step 5, numerical values have been defined for every descriptor. For example, for Black the value of A$ is 64.7 and the value of CT is 21.3. With this data the following summarizing table can be built:

MACBETH SUMMARIZING TABLE:

| **OVERALL OBJECTIVE**<br>"TO BUY THE BEST POSSIBLE VEHICLE MARIA CAN GET IN THE SECOND HAND MARKET OF COCHABAMBA" | | | | |
|---|---|---|---|---|
| CRITERIA:<br>WEIGHTS: | Economic<br>31.82 | Technical<br>25.00 | Comfort<br>13.64 | Environmental<br>29.55 |
| Black | 64.7 | 21.3 | 29.0 | 64.3 |
| Green | 35.3 | 29.5 | 29.0 | 35.7 |
| Grey | 0 | 34.4 | 41.9 | 35.7 |

## Step 8: Calculating a final result

Finally, an additive aggregation procedure is performed:

Black=31.82*64.7+25*21.3+13.64*29+29.55*64.3=4886.879=46.5%
Green=31.82*35.3+25*29.5+13.64*29+29.55*35.7=3311.241=31.5%
Grey=31.82*0+25*34.4+13.64*29+29.55*35.7=2310.495=22.0%

Accordingly, the Black is better than the Green, and the Green is better than the Grey.

## Comments on MacBeth

MacBeth software Version 1.1 is an algorithm to rank elements based on the data available on the MacBeth Matrix of Comparisons. This means that the Decision Maker must run the software once for every FPV. The software does not offer a final ranking of alternatives, but only a ranking of elements of every Matrix of Comparisons, which must be finally aggregated to obtain a final result.

A point in favor of MacBeth is that it works with indirect information. Pairwise comparisons are done considering difference of attractiveness rather than attractiveness itself. Moreover, the Decision Maker must develop impact descriptors in order to evaluate alternatives, rather than using scales of measurement with numerical values. Relative importance of criteria are elicitated by means of imagining and comparing fictitious alternatives which in fact represent criteria, rather than directly establishing relative weights of importance.

# IMPLEMENTING PROMETHEE METHOD: DECISION LAB

Decision Lab is a computer program developed for commercial purposes after the Promethee-Gaia Solution software. It is based on the *outranking* models Promethee I and II (J.P. Brans and B. Marechal).

## Step 1: Defining inputs for the performance table

To run this software, after defining criteria and alternatives, Maria has to input some data concerning alternatives. A Performance Table is required, in which alternatives are evaluated individually and for every criterion. To do so, Maria makes use of data available on the General Performance Table (Phase D). This allows her to see and remember relevant characteristics of every alternative.

Additionally, Maria has to imagine scales of measurement to be able to describe the particular condition of every alternative on each criterion. Scales of measurement used by Maria are shown next:

MEASUREMENT SCALES for EVALUATION

| **Criteria** | **Scale of Measurement** |
|---|---|
| Economic | from 0 to infinite |
| Technical | *New=9; Very Good=8; Good=7; More or Less Good=6; Acceptable=5; Somewhat Bad=4; Bad=3; Very Bad=2; Awful=1* |
| Aesthetics | *Excellent=9; Very Good=8; Good=7; More or Less Good=6; Indifferent=5; Somewhat Bad=4; Bad=3; Very Bad=2; Awful=1* |
| Comfort | *Excellent=9; Very Good=8; Good=7; More or Less Good=6; Indifferent=5; Somewhat Bad=4; Bad=3; Very Bad=2; Awful=1* |
| Environmental | *If fueled by natural gas=3; If fueled by gasoline=2; If fueled by diesel=1* |

Maria evaluates alternative performances with respect to each criterion, using measurement scales just defined. Results of evaluation are shown in the following table:

PROMETHEE PERFORMANCE TABLE:

| | ECONOMIC | TECHNICAL | AESTHETICS | COMFORT | ENVIRONMENTAL |
|---|---|---|---|---|---|
| Black | 12.340 | A(=5) | VG(=8) | MLG(=6) | Gas(=3) |
| Green | 15.260 | G(=7) | MLG(=6) | G(=7) | Gasoline(=2) |
| Grey | 17.835 | VG(=8) | I(=5) | G(=7) | Gasoline(=2) |

## Step 2: Defining criteria

Decision Lab requires the definition of attributes concerning criteria. Relevant aspects of this input procedure are shown next.

- *Category* is a group of actions, criteria or scenarios that are useful to identify related items and to perform sensitivity analysis on groups of criteria. It is not relevant to our example, but it could be useful for more complex problems.

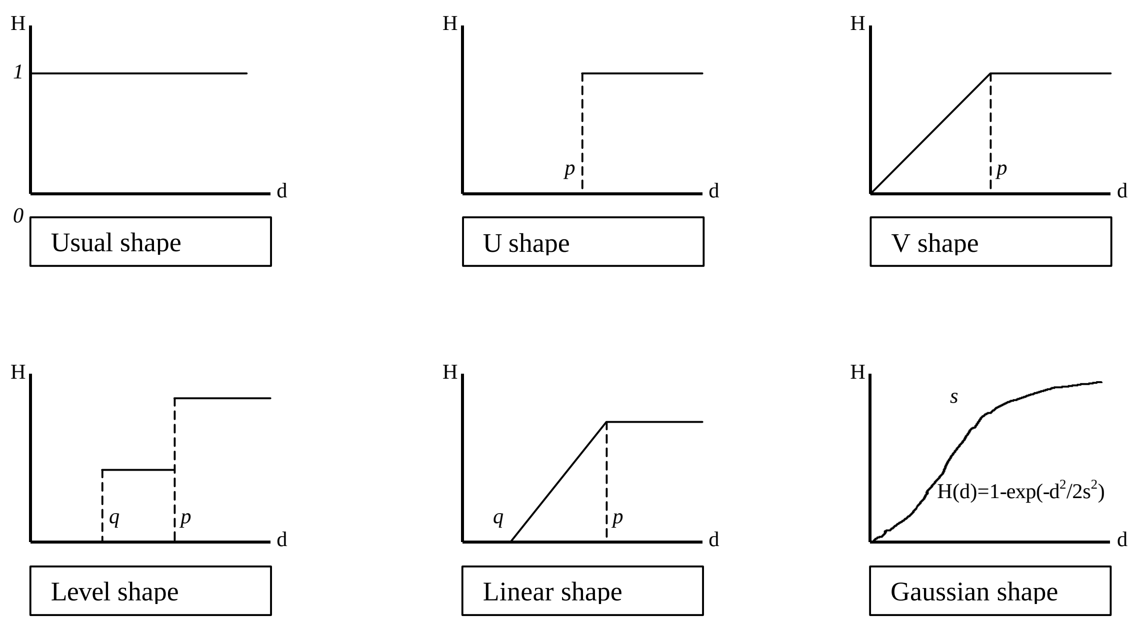
- *Min/Max* refers to whether the criterion should be maximized or minimized. For example, the criterion Economic has to be minimized (i.e. the smaller, the better) and the criterion Aesthetics has to be maximized.

- *Absolute weight* refers to a real number which does not depend on scales, describing the importance of a criterion. The sum of all weights, one for each criterion, must be equal to one.

- *Thresholds*: As it has been explained before (see Electre method), a threshold is a limit, set to take in to account compensatory effects in which good performances on some criteria cancel bad performances on other criteria. For example, an exceptionally good price together with a good technical performance may compensate a not completely satisfying aesthetics and comfort. With thresholds, limits are set for these compensatory effects.

- *Threshold unit* (see Data Input Table for Criteria) refers to whether the limit is set as a percentage or as a fixed amount on the performance scale. For example, Maria may consider a difference of $500 on the price of the vehicle, irrelevant. This means that given Maria's economic situation, a price of $10.000 may be equal to a price of $10.500. Her *Indifference Threshold (q)* is set at $500. But, if she has to compare a vehicle whose price is $10.000 with a vehicle whose price is $11.000, she will choose the cheapest one, because a $1.000 difference in price of is relevant for her. It is just too much difference. I.e. Maria's *Preference Threshold (p)* on Economic criterion is set at $1000.

- *Preference function*: Depending on the type of criterion, the Decision Maker's preference may vary according to the following functions:




For example, Maria chose *"Linear shape"* for Economic criterion because she is indifferent between $10.000 and $10.500 (Indifference Threshold *q* = $500), her preference increases linearly between $10.500 and $11.000 (Preference Threshold *p* = $1.000) and if the price difference is higher than $1.000, she surely prefers the cheapest one.

Next, a table is presented showing all data inputs for criteria, as required by Decision Lab:

DECISION LAB DATA INPUT TABLE for CRITERIA

| | **ECON** | **TECH** | **AEST** | **COMF** | **ENVI** |
|---|---|---|---|---|---|
| **Item** | Economic | Technical | Aesthetic | Comfort | Environmental |
| **Name** | Economic | Technical | Aesthetic | Comfort | Environmental |
| **Short Name** | Eco. | Tec. | Aes. | Com. | Env. |
| **Description** | n/a | n/a | n/a | n/a | n/a |
| **Enabled** | True | True | True | True | True |
| **Unit** | $ | n/a | n/a | n/a | n/a |
| **Decimals** | 3 | 3 | 3 | 3 | 3 |
| **Category** | (None) | (None) | (None) | (None) | (None) |
| **Threshold Unit** | Absolute | Absolute | Absolute | Absolute | Absolute |
| **Min/Max** | Minimize | Maximize | Maximize | Maximize | Maximize |
| **Absolute Weight** | 0.295 | 0.311 | 0.03 | 0.1 | 0.263 |
| **Preference Function** | Linear | Linear | Linear | Linear | Usual |
| **Scale** | (Numerical) | TECH | AEST | AEST | ENVI |
| **Indifferent Threshold** | 500 | 1 | 2 | 1 | n/a |
| **Preference Threshold** | 1000 | 2 | 3 | 2 | n/a |
| **Gaussian Threshold** | n/a | n/a | n/a | n/a | n/a |

## Step 3: Computing

This text is based on Brans, 1997.

Given the set of criteria G={$g_1,\ldots,g_j,\ldots,g_k$} and the set of alternatives A={$a_1,\ldots a_i,\ldots,a_n$}, the following performance table is built:

| | $g_1$ | $g_2$ | … | $g_j$ | … | $g_k$ |
|---|---|---|---|---|---|---|
| $a_1$ | $g_1(a_1)$ | $g_2(a_1)$ | … | $g_j(a_1)$ | … | $g_k(a_1)$ |
| $a_2$ | $g_1(a_2)$ | $g_2(a_2)$ | … | $g_j(a_2)$ | … | $g_k(a_2)$ |
| … | … | … | … | … | … | … |
| $a_i$ | $g_1(a_i)$ | $g_2(a_i)$ | … | $g_j(a_i)$ | … | $g_k(a_i)$ |
| … | … | … | … | … | … | … |
| $a_n$ | $g_1(a_n)$ | $g_2(a_n)$ | … | $g_j(a_n)$ | … | $g_k(a_n)$ |

where $g_j(a_i)$ represents alternative $a_i$*'s* performance with respect to criterion $g_j$

Additionally, the Decision Maker has to set weights of importance $p_j$ of criteria:

| $g_1$ | $g_2$ | … | $g_j$ | … | $g_k$ |
|---|---|---|---|---|---|
| $p_1$ | $p_2$ | … | $p_j$ | … | $p_k$ |

- The higher is the weight, the more important is the criterion
- $\sum_{j=1..k} p_j = 1$

Depending on the Decision Maker's preferences, a preference function for each criterion has to be found. As said before, there are six types of preference functions: Usual type, U shape, V shape, Level criterion, V shape and Gaussian.

Preference functions are defined as follows:

$$F_j(a,b)=F_j[d_j(a,b)],\ \ a,b\in A$$

where, j is the characteristic index of $g_j$
$d_j(a,b)=g_j(a)-g_j(b)$
$0 \le F_j(a,b) \le 1$

Then, the *aggregated preference index* $\pi(a,b)$ is calculated:

$$\pi(a,b)=\sum_{j=1..k} F_j(a,b)\, p_j\text{ , being } 0\le\pi(a,b) \le 1 \text{ and } \pi(a,a)=0$$

$\pi(a,b)$ expresses how and which degree $a$ is preferred to $b$ over all the criteria
$\pi(a,b)$ close to 0 means a weak global preference of $a$ over $b$
$\pi(a,b)$ close to 1 means a strong global preference of $a$ over $b$

Then, *outranking flows* are calculated as follows:

*Positive outranking flow*, which expresses how an alternative $a$ outranks the others:

$$\varphi^+(a)=(1/(n-1)) \sum_{x\in A}\pi(a,x),$$

*Negative outranking flow*, which expresses how an alternative $a$ is outranked by others:

$$\varphi^-(a)=(1/(n-1)) \sum_{x\in A}\pi(x,a),$$

A partial ranking of alternatives can be deduced from the positive and negative outranking flows:

$$\begin{cases} aPb & \text{if } \begin{cases} \varphi^{+}(a) \geq \varphi^{+}(b) \text{ and } \varphi^{-}(a) < \varphi^{-}(b) \\ \varphi^{+}(a) > \varphi^{+}(b) \text{ and } \varphi^{-}(a) = \varphi^{-}(b) \end{cases} \\ aIb & \text{if } \varphi^{+}(a) = \varphi^{+}(b) \text{ and } \varphi^{-}(a) = \varphi^{-}(b) \\ aRb & \text{otherwise} \end{cases}$$

where P, I and R are “strict preference”, “indifference” and “incomparability” relations respectively.

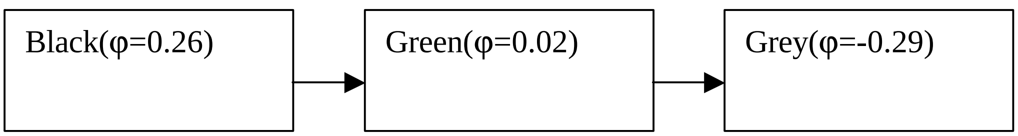
To calculate complete rankings the *net outranking flow* is used:

$$\varphi(a) = \varphi^{+}(a) - \varphi^{-}(a)$$

$$\begin{cases} aPb & \text{if } \varphi(a) > \varphi(b) \\ aIb & \text{if } \varphi(a) = \varphi(b) \end{cases}$$

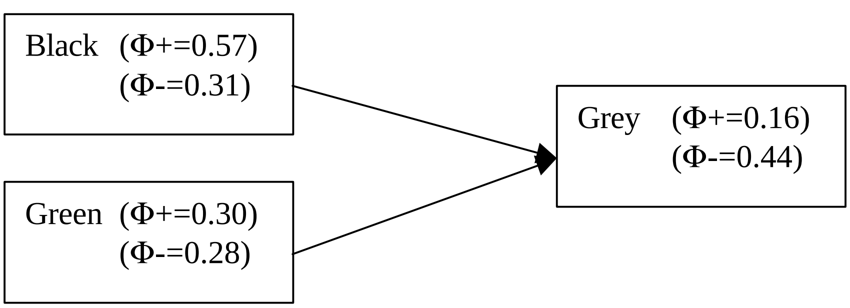
## Step 4: Getting results from Decision Lab

*Complete ranking* (Promethee I):




Complete ranking means that incomparability is not allowed to happen among alternatives. In this case, Black is better than Green, and Green is better than Grey.

*Partial ranking* (Promethee II):




In the case of partial ranking, *strict preference, indifference* and *incomparability* are allowed. This means that in some cases, it may happen that two alternatives are barely equal (i.e comparable), or just incomparable. For example, given that the criterion “Economic” and the criterion “Environmental” are both strong, a vehicle with a very good price but very polluting can be difficult to compare with an environment-friendly but very expensive vehicle. Similarly, given that "Aesthetics" is not an important criterion, Maria can be indifferent between two similar vehicles, one aesthetically much nicer than the other. In our example, Green and Black are both better than Grey. Based on results of partial ranking, a more careful analysis of preferences may be suggested, perhaps considering some other criteria, before defining the final preference between Green and Black.

### Comments on the Promethee

Promethee method is quite similar to ELECTRE, except: 1) the definition of shapes of preference functions, which can be overwhelmingly difficult for a Decision Maker, and 2) Promethee does not allow for veto.

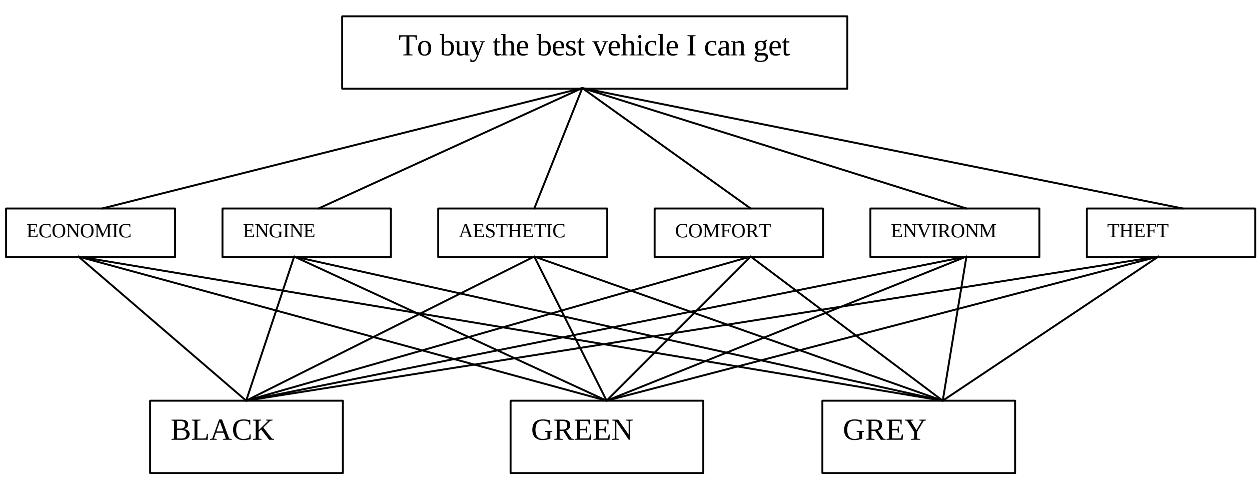
## IMPLEMENTING ANALYTIC HIERARCHY PROCESS: EXPERT CHOICE

The Analytical Hierarchy Process (AHP) is a decision making process developed by Thomas Saaty. It is based on series of one-to-one comparisons throughout a hierarchy to arrive at overall priorities for alternatives. Expert Choice is the software developed based on of this model.

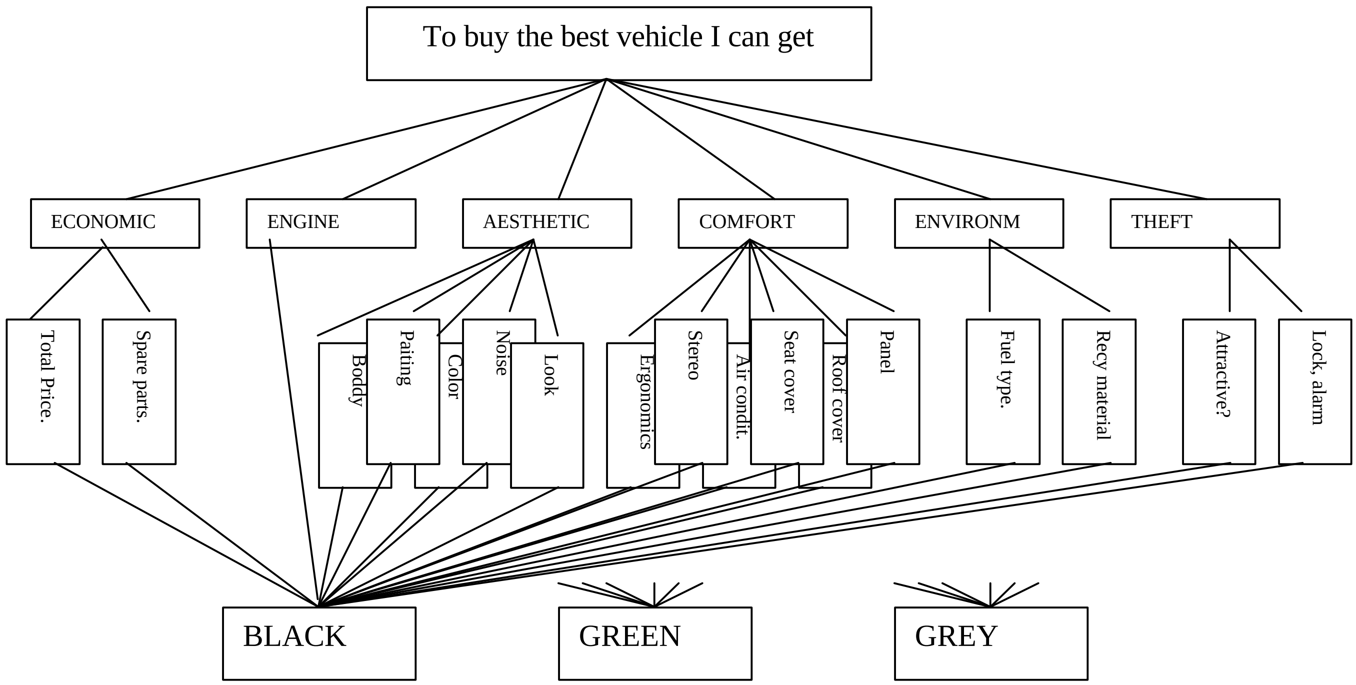
### Step 1: Building the hierarchy

Based on the data available from Phases Q, C, I, A and D, Maria was able to imagine more than one hierarchical structure:

OPTION 1:



<u>OPTION 2</u>:




Due to software constraints (we got the trial version only), Option1 is chosen to exemplify the use of this method. We want to stress the fact that it would be much more interesting to use the second hierarchy (Option2) because it possesses the level in which those different aspects of every criterion are considered. Taking into account only three levels (objective, criteria and alternatives) implies that the Decision Maker has to make "inside of the mind" weighting processes to evaluate the relative importance of the elements of the third level in Option1, and their correspondent assessment for every alternative. For example, in regard to the "Economic" criterion, the Decision Maker has to evaluate "inside of the mind" the relative importance of "price" with respect to "spare-parts" and vice versa. This process can be difficult or misleading.

## Step 2: Making pairwise comparisons

Making pairwise comparisons means making abstraction of the overall state of things, to concentrate only on two elements of a given level of the hierarchy. For example, we can compare two alternatives, let's say the *black* and the *green* with respect and only with respect to the criterion *comfort*. The Decision Maker must put aside in his mind any source of bias concerning consideration of other criteria and other alternatives in the pair-wise comparison process. This level of abstraction can be difficult for some people, and it is a task of the analyst to extract sincere and bias-free answers from the Decision Maker.

Some questions should be put for every pairwise comparison:

Is *black* preferred to *green*, concerning *comfort*?
If yes:
How much preferable is *black* compared to *green*, concerning *comfort*?
If no:
How much preferable is *green* compared to *black*, concerning *comfort*?
or:
Is the criterion *Engine* more important than the criterion *Comfort*, concerning the *overall objective* we want to achieve?
If yes:
How much more important is the criterion *Engine* compared to the criterion *Comfort*, concerning the *overall objective* we want to achieve?
If no:
How much more important is the criterion *Comfort* compared to the criterion *Engine*, concerning the *overall objective* we want to achieve?

To be able to answer questions related to the degree of preference, importance or likelihood, the following comparison scale must be used:

AHP COMPARISON SCALE:

| **Intensity of importance** | **Explanation** | |
|---|---|---|
| 1 | Equal importance | Two activities contribute equally to the objective |
| 3 | Weak importance of one over another | Experience and judgment slightly favor one activity over another |
| 5 | Essential or strong importance | Experience and judgment strongly favor one activity over another |
| 7 | Very strong importance | An activity is favored very strongly over another; its dominance is demonstrated in practice |
| 9 | Absolute importance | The evidence favoring one activity over another is of highest possible order of affirmation |
| 2, 4, 6, 8 | Intermediate values between adjacent scale values | When compromise is needed |
| Reciprocals of above nonzero | If activity i has one of the above nonzero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i | A reasonable assumption |
| Rationales | Ratios arising from the scale | If consistency were to be forced by obtaining n numerical values to span the matrix |

Back to our example, it is time for Maria to make the pairwise comparisons. Maria has then to compare every pair of elements of every level, by means of the sentences in the comparison scale just shown.

These are the results of the pair-wise comparisons for all elements and levels of the hierarchy:

With respect to the overall objective: *"TO BUY THE BEST VEHICLE I CAN GET":*

| CRITERIA | LINGUISTIC DESCRIPTION OF PREFERENCE | CRITERIA | VALUE |
|---|---|---|---|
| ECONOMIC | is EQUALLY as IMPORTANT as | ENGINE | 1 |
| ECONOMIC | is VERY STRONGLY to EXTREMELY more IMPORTANT than | AESTHETI | 8 |
| ECONOMIC | is VERY STRONGLY more IMPORTANT than | COMFORT | 7 |
| ECONOMIC | is EQUALLY as IMPORTANT as | ENVIRONM | 1 |
| ECONOMIC | is VERY STRONGLY more IMPORTANT than | THEFT | 7 |
| ENGINE | is VERY STRONGLY to EXTREMELY more IMPORTANT than | AESTHETI | 8 |
| ENGINE | is VERY STRONGLY to EXTREMELY more IMPORTANT than | COMFORT | 8 |
| ENGINE | is EQUALLY as IMPORTANT as | ENVIRONM | 1 |
| ENGINE | is VERY STRONGLY to EXTREMELY more IMPORTANT than | THEFT | 8 |
| COMFORT | is EQUALLY to MODERATELY more IMPORTANT than | AESTHETI | 2 |
| ENVIRONM | is VERY STRONGLY more IMPORTANT than | AESTHETI | 7 |
| THEFT | is MODERATELY more IMPORTANT than | AESTHETI | 3 |
| ENVIRONM | is VERY STRONGLY more IMPORTANT than | COMFORT | 7 |
| COMFORT | is EQUALLY as IMPORTANT as | THEFT | 1 |
| ENVIRONM | is MODERATEY to STRONGLY more IMPORTANT than | THEFT | 6 |

With respect to the criteria: *"ECONOMIC ASPECTS":*

| CRITERIA | LINGUISTIC DESCRIPTION OF PREFERENCE | CRITERIA | VALUE |
|---|---|---|---|
| BLACK | is MODERATELY to STRONGLY more IMPORTANT than | GREEN | 4 |
| BLACK | is EXTREMELY more PREFERABLE than | GREY | 9 |
| GREEN | is MODERATELY to STRONGLY more IMPORTANT than | GREY | 4 |

With respect to the criteria: *"ENGINE CONDITION AFTER COMPLETE REPAIRING":*

| CRITERIA | LINGUISTIC DESCRIPTION OF PREFERENCE | CRITERIA | VALUE |
|---|---|---|---|
| GREEN | is STRONGLY more PREFERABLE than | BLACK | 5 |
| GREY | is STRONGLY to VERY STRONGLY more PREFERABLE than | BLACK | 6 |
| GREY | is MODERATELY more PREFERABLE than | GREEN | 3 |

With respect to the criteria: *"AESTHETICS":*

| CRITERIA | LINGUISTIC DESCRIPTION OF PREFERENCE | CRITERIA | VALUE |
|---|---|---|---|
| BLACK | is STRONGLY more PREFERABLE than | GREEN | 5 |
| BLACK | is VERY STRONGLY to EXTREMELY more PREFERABLE than | GREY | 8 |
| GREEN | is MODERATELY more PREFERABLE than | GREY | 3 |

With respect to the criteria: *"COMFORT":*

| CRITERIA | LINGUISTIC DESCRIPTION OF PREFERENCE | CRITERIA | VALUE |
|---|---|---|---|
| BLACK | is EQUALLY as IMPORTANT as | GREEN | 1 |
| BLACK | is EQUALLY as IMPORTANT as | GREY | 1 |
| GREEN | is EQUALLY as IMPORTANT as | GREY | 1 |

With respect to the criteria: *"ENVIRONMENTAL ASPECTS":*

| CRITERIA | LINGUISTIC DESCRIPTION OF PREFERENCE | CRITERIA | VALUE |
|---|---|---|---|
| BLACK | is VERY STRONGLY to EXTREMELY more IMPORTANT than | GREEN | 8 |
| BLACK | is VERY STRONGLY to EXTREMELY more PREFERABLE than | GREY | 8 |
| GREY | is EQUALLY to MODERATELY more PREFERABLE than | GREEN | 2 |

With respect to the criteria: *"RISK OF THEFT":*

| CRITERIA | LINGUISTIC DESCRIPTION OF PREFERENCE | CRITERIA | VALUE |
|---|---|---|---|
| BLACK | is EQUALLY as IMPORTANT as | GREEN | 1 |
| BLACK | is EQUALLY as IMPORTANT as | GREY | 1 |
| GREEN | is EQUALLY as IMPORTANT as | GREY | 1 |

These pairwise comparisons will be recorded for computation purposes in the form of matrices. An example of the comparison matrix for the criteria level is given as follows:

| | ECON | ENGI | AEST | COMF | ENVI | STEA |
|---|---|---|---|---|---|---|
| **ECONOMIC** | 1 | 1 | 8 | 7 | 1 | 7 |
| **ENGINE** | | 1 | 8 | 8 | 1 | 8 |
| **AESTHETICS** | | | 1 | | | |
| **COMFORT** | | | 2 | 1 | | 1 |
| **ENVIRONMENT** | | | 7 | 7 | 1 | 6 |
| **RISK OF THEFT** | | | 3 | | | 1 |

## Step 3: Computing

Once all pairwise comparison matrices are built, there are three computation steps that are to be performed:

- Checking consistency of every comparison matrix
- Applying the eigenvector method to compute the weights
- Aggregating the weights to determine a ranking of alternatives

### *Checking consistency*

Consider a comparison case in which a Decision Maker states that alternative 1 is twice as important as alternative 2 and alternative 2 is twice as important as alternative 3. If the same Decision Maker states that alternative 1 is three times as important as alternative 3, then the comparison is not perfectly consistent because alternative 1 should be four times as important as alternative 3 in view of two comparisons made earlier.

In general, what we mean by being consistent is that when we have a basic amount of raw data, all other data can be logically deduced from it. For example, in the following matrix,

| *B* | *E* | *F* | *G* | *H* |
|---|---|---|---|---|
| *E* | 1 | 2 | 1 | 3 |
| *F* | 1/2 | 1 | 1/2 | 3/2 |
| *G* | 1 | 2 | 1 | 3 |
| *H* | 1/3 | 2/3 | 1/3 | 1 |

by filling in the first row, all the other positions on the matrix can be deduced. This matrix is said to be perfectly consistent.

In fact, the model checks for consistency by using the concept of the "eigenvector" and "eigenvalue". An eigenvector w of a matrix A is a characteristic vector such that $Aw=\lambda w$. The values of $\lambda$ corresponding to such a w are called the eigenvalues of A. Thus, w would be an eigenvector if it is a nontrivial solution of $(A-\lambda I)w=0$ for some number $\lambda$. The components of w constitute a set of solutions of a homogeneous linear system with matrix $A-\lambda I$. Such a system, in fact, has the trivial solution $w_1=w_2=\ldots=w_n=0$ where $w=(w_1,\ldots,w_n)$. But in order to have a nontrivial solution, the matrix $A-\lambda I$ must be singular, i.e., its determinant $\det(A-\lambda I)$ should be zero. This determinant is an $n^{th}$ degree polynomial in $\lambda$. It has the form $\lambda^n-a_1\lambda^{n-1}+\ldots+(-1)\det(A)$ and is called the characteristic polynomial of A. The condition that the determinant should equal zero leads to an $n^{th}$ degree equation called the characteristic equation of A which is identically zero if $\lambda$ is replaced by A, thus yielding a matrix equation. The roots $\lambda_i$ for i=1,…,n, of the characteristic equation $\det(A-\lambda I)=0$ are the desired eigenvalues. The fundamental theorem of algebra assures the existence of n roots for a polynomial equation of degree n. The eigenvectors are obtained by solving the corresponding systems of equations, $Av_i=\lambda_i v_i$ (Saaty, 1980).

We denote by $a_{ij}$ the number indicating the strength of $C_i$ when compared with $C_j$. The matrix of these numbers is denoted by A. This matrix is reciprocal, i.e., $a_{ij}=1/a_{ji}$. If our judgment is perfect in all comparisons, then $a_{ik}=a_{ij}*a_{jk}$ for all i,j,k and we call the matrix A '*consistent*'. If $\lambda_1,\ldots,\lambda_n$ are the eigenvalues of A, and if $a_{ii}=1$ for all i, then $(\lambda_1+\ldots+\lambda_n)=n$. The latest statement comes from matrix theory and its discussion is not considered in this work. Saaty states that, in the consistent case, n is the largest eigenvalue of A, denoted by $\lambda_{max}$.

Another concept of matrix theory, which discussion we omit here, is that if one changes the entries $a_{ij}$ of a positive reciprocal matrix A by small amounts, then the eigenvalues change by small amounts. Then, if A is consistent, small variations of the $a_{ij}$ keep the largest eigenvalue, $\lambda_{max}$, close to n. Thus, if A is the matrix of pair-wise comparison values, in order to find the priority vector, we must find the vector which satisfies $Aw=\lambda_{max}w$. Since small changes in $a_{ij}$ imply small changes in $\lambda_{max}$, the deviation of the latter from n is a measure of consistency. Saaty (1980) takes the value $(\lambda_{max}-n)/(n-1)$ as a measure of consistency, which he calls consistency index (C.I.). How bad our consistency is in a given problem may be estimated by comparing our C.I. with a C.I. of a matrix of the same size which entries and reciprocals in the reverse positions have been randomly chosen. This consistency index is called random index (R.I.). A table with R.I. for matrixes of several orders follows:

| Order | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| R.I. | 0 | 0 | 0.58 | 0.90 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 | 1.51 | 1.48 | 1.56 | 1.57 | 1.59 |

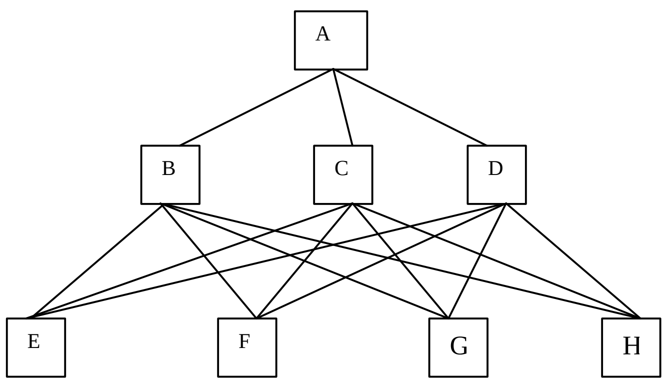
The ratio of C.I. to the average R.I. for the same order matrix is called the consistency ratio (C.R.). If the consistency ratio surpasses 0.1, a new input matrix must be generated because of inconsistency. If the consistency ratio is in the neighborhood of 0.1, we can be satisfied with our judgment.

### *Applying the eigenvector method to compute the weights*

Let us consider the elements $C_1,\ldots,C_n$ of some level in the hierarchy. We wish to find their weights of influence, $w_1,\ldots,w_n$, on some element in the next level. As described earlier, our basic tool is a matrix of numbers, representing our judgment of pairwise comparisons. We calculate the weights by calculating the eigenvector of the pairwise comparison matrix. Once we have the pairwise comparison matrices, the next step is to find the eigenvector of each matrix, which represents the relative weights of the alternatives over the given criteria. The eigenvector with the largest eigenvalue is chosen to provide the priorities (Saaty, 1980).

### *The Additive Aggregation Procedure to obtain a final result*

In this stage we already have all the relative weights of all alternatives over all the criteria in all levels of the hierarchy. So, what we now need to calculate is the importance of alternatives (bottom level) with respect to the overall objective (top level). Consider the following example:




Pairwise comparisons results are placed in matrices:

| $A$ | $B$ | $C$ | $D$ | *Weights* |
|---|---|---|---|---|
| $B$ | $1$ | $a_{bc}$ | $a_{bd}$ | $W_{B/A}$ |
| $C$ | $1/a_{bc}$ | $1$ | $a_{cd}$ | $W_{B/A}$ |
| $D$ | $1/a_{bd}$ | $1/a_{cd}$ | $1$ | $W_{D/A}$ |

| $C$ | $E$ | $F$ | $G$ | $H$ | $W$ |
|---|---|---|---|---|---|
| $E$ | $1$ | $a_{ef/c}$ | $a_{eg/c}$ | $a_{eh/c}$ | $W_{E/C}$ |
| $F$ | $1/a_{ef/c}$ | $1$ | $a_{fg/c}$ | $a_{fh/c}$ | $W_{F/C}$ |
| $G$ | $1/a_{eg/c}$ | $1/a_{fg/c}$ | $1$ | $1/a_{gh/c}$ | $W_{G/C}$ |
| $H$ | $1/a_{eh/c}$ | $1/a_{fh/c}$ | $1/a_{gh/c}$ | $1$ | $W_{H/C}$ |

| $B$ | $E$ | $F$ | $G$ | $H$ | $W$ |
|---|---|---|---|---|---|
| $E$ | $1$ | $a_{ef/b}$ | $a_{eg/b}$ | $a_{eh/b}$ | $W_{E/B}$ |
| $F$ | $1/a_{ef/b}$ | $1$ | $a_{fg/b}$ | $a_{fh/b}$ | $W_{F/B}$ |
| $G$ | $1/a_{eg/b}$ | $1/a_{fg/b}$ | $1$ | $a_{gh/b}$ | $W_{G/B}$ |
| $H$ | $1/a_{eh/b}$ | $1/a_{fh/b}$ | $1/a_{gh/b}$ | $1$ | $W_{H/B}$ |

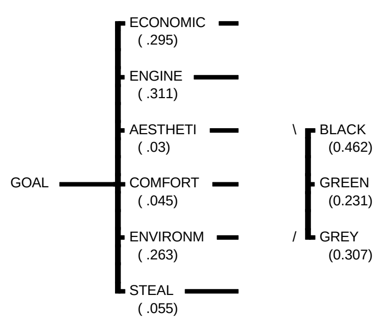
| $D$ | $E$ | $F$ | $G$ | $H$ | $W$ |
|---|---|---|---|---|---|
| $E$ | $1$ | $a_{ef/d}$ | $a_{eg/d}$ | $a_{eh/d}$ | $W_{E/D}$ |
| $F$ | $1/a_{ef/d}$ | $1$ | $a_{fg/d}$ | $a_{fh/d}$ | $W_{F/D}$ |
| $G$ | $1/a_{eg/d}$ | $1/a_{fg/d}$ | $1$ | $a_{gh/d}$ | $W_{G/D}$ |
| $H$ | $1/a_{eh/d}$ | $1/a_{fh/d}$ | $1/a_{gh/d}$ | $1$ | $W_{H/D}$ |

Once the relative weights (i.e. $W_{B/A}$, $W_{E/B}$, $W_{C/A}$, $W_{E/C}$, $W_{D/A}$, $W_{E/D}$, …) have been calculated by the eigenvector method and the consistency has been checked by the eigenvalue method, we can aggregate the overall preference as follows:

Overall weight of alternative $E=W_{B/A}*W_{E/B}+W_{C/A}*W_{E/C}+W_{D/A}*W_{E/D}$
Overall weight of alternative $F=W_{B/A}*W_{F/B}+W_{C/A}*W_{F/C}+W_{D/A}*W_{F/D}$
Overall weight of alternative $G=W_{B/A}*W_{G/B}+W_{C/A}*W_{G/C}+W_{D/A}*W_{G/D}$
Overall weight of alternative $H=W_{B/A}*W_{H/B}+W_{C/A}*W_{H/C}+W_{D/A}*W_{H/D}$

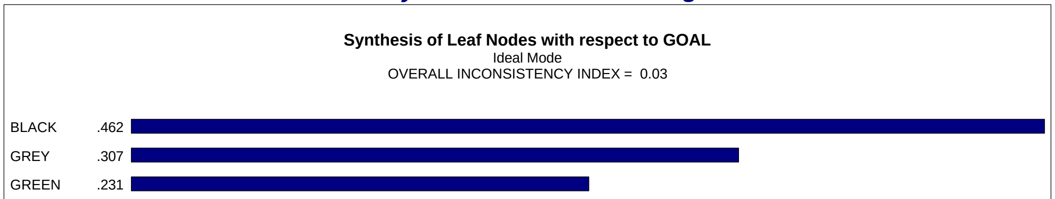
## Step 4: Getting results from Expert Choice

We show Expert Choice's output of our problem.




| Abbreviation | Definition |
|---|---|
| GOAL | |
| AESTHETI | AESTHETICS |
| BLACK | BLACK PICK-UP |
| COMFORT | COMFORT |
| ECONOMIC | ECONOMIC ASPECTS |
| ENGINE | ENGINE CONDITION AFTER COMPLETE REPAIRING |
| ENVIRONM | ENVIRONMENTAL ASPECTS |
| GREEN | GREEN PICK-UP |
| GREY | GREY PICK-UP |
| STEAL | RISK OF STEAL |

***To buy the best vehicle I can get***




| Abbreviation | Definition |
|---|---|
| BLACK | BLACK PICK-UP |
| GREY | GREY PICK-UP |
| GREEN | GREEN PICK-UP |

***Trial Use Only***

## Comments on AHP

With AHP it is possible to consider any number of levels in the hierarchy. It means that it is possible to avoid "*inside of the mind comparison processes*", which can be difficult to make or even misleading, avoiding in this way direct elicitation of weights of relative importance of criteria. With AHP it is possible to decompose the decision making process until the Decision Maker is sure about the feasibility and easiness of the comparisons s/he got to make. It can give clarity to the structuring phase of the process.

When comparing alternatives regarding the criterion *Economic*, the matrix made with bias-free and sincere pair-wise comparisons was inconsistent: The mental comparison process was done as follows:

⇨Question: Concerning the criterion "Economics": Is *black* better than *green*?
⇨Answer: Yes, *black* is better than *green* because *black* costs $12.340 and *green* costs $15.260. Besides, *green*'s spare-parts seems to be more expensive.
⇨ Question: How better is *black* compared to *green*?
⇨ Answer: *Black* seems to be absolutely better than *green*, so I would put "*extreme*" on the matrix.
⇨ Question: Now, concerning the same criterion: Is *green* better than *grey*?
⇨ Answer: Yes, given that *green* is $2.575 cheaper that *grey*, I would never choose *grey* concerning the criterion "Economics" only. I can not say, for instance, that *green* is *moderately better* than *grey* because money is a scarce resource and I must buy the

cheapest alternative when the only criterion to consider is the economic aspect. For that reason, *green* is extremely better than *grey* concerning criterion "Economics".
⇨ Question: Is *black* better than *grey*?
⇨ Answer: Yes, given that *black* is the cheapest alternative and *grey* is the most expensive alternative, concerning economic aspects, *black* is extremely better than *grey*.

So the following matrix with a high degree if inconsistency was obtained:

| ECONOMIC | black | green | grey |
|---|---|---|---|
| black | 1 | 9 | 9 |
| green | | 1 | 9 |
| grey | | | 1 |

To achieve consistency, some changes had to be made in the matrix taking in to account the amount of difference in prices.

| ECONOMIC | black | green | grey |
|---|---|---|---|
| black | 1 | 4 | 9 |
| green | | 1 | 4 |
| grey | | | 1 |

This matrix is consistent, but it does not express the decision-maker true feelings.

## IMPLEMENTING NAIADE

NAIADE (Novel Approach to Imprecise Assessment and Decision Environments) is a discrete multicriteria method authored by Giuseppe Munda. NAIADE allows consideration of either crisp, stochastic or fuzzy measurements.

### Step 1: Defining criteria

Within NAIADE, five types of criteria can be defined:

- Qualitative
- Quantitative crisp
- Quantitative fuzzy
- Quantitative stochastic
- Quantitative numeric and fuzzy

In what follows, we show the required data input for every type of criteria:

**Qualitative criteria** (i.e. a subjective statement like "*the comfort of this car is very good*")
⇨Goal type: Choose maximize/minimize
⇨Setting the *credibility level (index)* for distance between *moderate* and *good*: The credibility index of each criterion is a number from 0 (definitely non-credible) to 1 (definitely credible) expressed by means of preference relations defined as the statements *much better* ($\mu_{>>}$), *better* ($\mu_{>}$), *approximately equal* ($\mu_{\cong}$), *equal* ($\mu_{==}$), *worse* ($\mu_{<}$) and *much worse* ($\mu_{<<}$). Given the

linguistic variables *perfect* ($\mu_p$), *very good* ($\mu_{vg}$), *good* ($\mu_g$), *more or less good* ($\mu_{mlg}$), *moderate* ($\mu_m$), *more or less bad* ($\mu_{mlb}$), *bad* ($\mu_b$), *very bad* ($\mu_{vb}$) and *extremely bad* ($\mu_{eb}$) and their corresponding membership functions, NAIADE asks for expressing a credibility index of the following statements:

"GOOD" is much better than "MODERATE"
"GOOD" is better than "MODERATE"
"GOOD" is approximately equal to "MODERATE"
"GOOD" is equal than "MODERATE"

The credibility index of each statement is associated viz-a-viz to a corresponding crossover value. Crossover values are shown by NAIADE in the input data window.

The following constraints must apply:

1) $\mu_{>>}(d) = \mu_{<<}(-d)$ and $\mu_{>}(d) = \mu_{<}(-d)$: *worse* and *much worse* are mirrors of *better* and *much better*.
2) $C_{=}<C_{\cong}<C_{>}<C_{>>}$: The Decision Maker has to set credibility levels according to this rule. Note that credibility levels are function of crossover values.

In order to set the credibility levels, it is recommended to use the default values given by NAIADE:

| | |
|---|---|
| Credibility Level for $\mu_{>>}$ | 0.38 |
| Credibility Level for $\mu_{>}$ | 0.58 |
| Credibility Level for $\mu_{\cong}$ | 0.33 |
| Credibility Level for $\mu_{=}$ | 0 |

**Quantitative crisp criteria** (i.e. a crisp number like "*the price of this car is $17.250*" )
⇨Goal type: Choose maximize/minimize
⇨Establishing the plot of fuzzy preference relations: Despite the crisp nature of criteria, the Decision Maker may consider the preference relation to be fuzzy. The procedure to specify parameters of the fuzzy preference relation is as follows:

✓ Establishing *limits* for fuzzy preference relation functions: Each fuzzy preference relation function (i.e. *much better* ($\mu_{>>}$), *better* ($\mu_{>}$), *approximately equal* ($\mu_{\cong}$), *equal* ($\mu_{==}$), *worse* ($\mu_{<}$) and *much worse* ($\mu_{<<}$)) must be defined within a domain. The left limit is established at zero. The Decision Maker has to set the right limit according to the magnitude order of data.

✓ Establishing *crossover values* for every fuzzy preference relation function: *Crossover values* ($C_{==}$, $C_{\cong}$, $C_{>}$, $C_{>>}$) are points where the fuzzy preference relation equals 0.5.

**Quantitative fuzzy criteria** (i.e. a fuzzy number like “the gas consumption of this car is about 8 km/liter” )
⇨Goal type: Choose maximize/minimize
⇨Establishing the plot of fuzzy preference relations: Idem Quantitative crisp criteria.

In order to set the crossover values, it is recommended to use proportions to default values given by NAIADE:

| | |
|---|---|
| Plot limit | 10 |
| $C_{>>}$ | 4 |
| $C_{>>}$ | 3 |
| $C_{\cong}$ | 2 |
| $C_{=}$ | 1 |

**Quantitative stochastic criteria** (i.e. a probability density function that allows expressing “*these tires last in average 30.000 km*”)

⇨ Idem Quantitative fuzzy criteria

**Quantitative numeric and fuzzy (**i.e. a given criterion can be either quantitative crisp or quantitative fuzzy**)**

⇨ Idem Quantitative fuzzy criteria

## Step 2: Defining alternatives

This is an easy step. The name and a description of every alternative is entered in the performance table shown by NAIADE.

After phase A, Maria entered three choices:

- ☑ The black
- ☑ The green
- ☑ The grey

## Step 3: Evaluating Alternatives: filling in the performance table

- NAIADE interacts with the Decision Maker through a table. This table, known as Performance Table, is shown as the principal element of the principal window of the software NAIADE. The table’s first row (i.e. column labels) is composed by the alternatives. The first column (i.e. row labels) is composed by criteria. The Decision Maker’s task is to fill the Performance Table with performances of each alternative with respect to each criterion.

In what follows, we show the way to establish performances according to every type of criteria.

**Qualitative criteria.** When a qualitative criterion is considered, alternatives are evaluated by means of 9 pre-defined linguistic variables, whose membership functions are shown next:

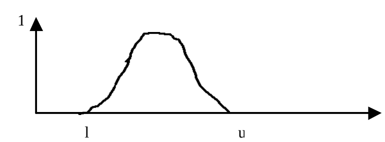
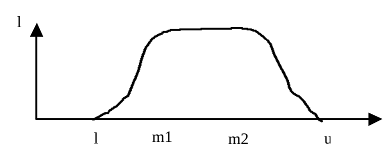
| **Linguistic variable** | **Membership functions** |
|---|---|
| *Perfect* | $\mu_p(x)=\begin{cases}1 \text{ for } x=1\\ 0 \text{ for } x\neq 1\end{cases}$ |
| *Very good* | $\mu_{vg}(x)=\begin{cases}4((x-0.8)/0.2)^4 \text{ for } 0.8\leq x\leq 0.9\\ [1-2((1-x)/0.2)^2)^2 \text{ for } 0.9\leq x\leq 1\\ 0 \text{ otherwise}\end{cases}$ |
| *Good* | $\mu_g(x)=\begin{cases}0 \text{ otherwise}\\ 2((x-0.6)/0.2)^2 \text{ for } 0.6\leq x\leq 0.7\\ 1-2((x-0.8)/0.2)^2 \text{ for } 0.7\leq x\leq 0.9\\ 2((1-x)/0.2)^2 \text{ for } 0.9\leq x\leq 1\end{cases}$ |
| *More or less good* | $\mu_{mlg}(x)=\begin{cases}2^{0.5}((x-0.5)/0.2) \text{ for } 0.5\leq x\leq 0.6\\ ((1-2((x-0.7)/0.2)^2)^{0.5} \text{ for } 0.6\leq x\leq 0.8\\ 2^{0.5}((0.9-x)/0.2) \text{ for } 0.8\leq x\leq 0.9\\ 0 \text{ otherwise}\end{cases}$ |
| *Moderate* | $\mu_m(x)=\begin{cases}2((x-0.3)/0.2)^2 \text{ for } 0.3\leq x\leq 0.4\\ 1-2((x-0.5)/0.2)^2 \text{ for } 0.4\leq x\leq 0.6\\ 2((0.7-x)/0.2)^2 \text{ for } 0.6\leq x\leq 0.7\\ 0 \text{ otherwise}\end{cases}$ |
| *More or less bad* | $\mu_{mlb}(x)=\begin{cases}2^{0.5}((x-0.1)/0.2) \text{ for } 0.1\leq x\leq 0.2\\ ((1-2((x-0.3)/0.2)^2)^{0.5} \text{ for } 0.2\leq x\leq 0.4\\ 2^{0.5}((0.5-x)/0.2) \text{ for } 0.4\leq x\leq 0.5\\ 0 \text{ otherwise}\end{cases}$ |
| *Bad* | $\mu_b(x)=\begin{cases}0 \text{ otherwise}\\ 2(x/0.2)^2 \text{ for } 0\leq x\leq 0.1\\ 1-2((x-0.2)/0.2)^2 \text{ for } 0.1\leq x\leq 0.3\\ 2((0.4-x)/0.2)^2 \text{ for } 0.3\leq x\leq 0.4\end{cases}$ |

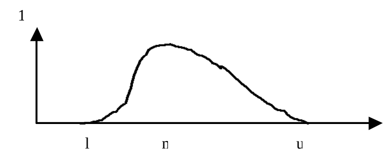
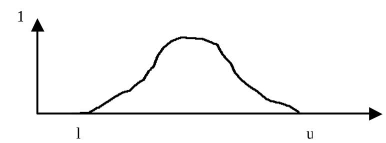
| | |
|---|---|
| *Very bad* | $\mu_{vb}(x)=\begin{cases}4((0.2-x)/0.2)^4 \text{ for } 0.1\le x\le 0.2\\ [1-2(x/0.2)^2)^2 \text{ for } 0\le x\le 0.1\\ 0 \text{ otherwise}\end{cases}$ |
| *Extremely bad* | $\mu_{eb}(x)=\begin{cases}1 \text{ for } x=1\\ 0 \text{ for } x\ne 1\end{cases}$ |

The Decision Maker must use one of these linguistic variables in order to describe alternative performances related to a subjective criterion. NAIADE uses the corresponding membership function in order to calculate overall preferences.

**Quantitative crisp criteria.** The task of the Decision Maker is to enter the perfectly known and exact value corresponding to the evaluation of a given alternative on a given criterion.

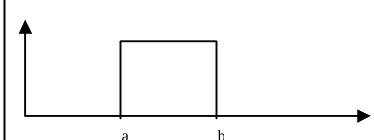
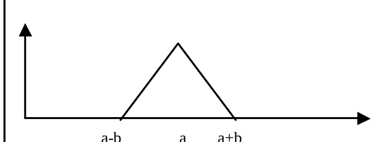
**Quantitative fuzzy criteria.** When dealing with criteria with fuzzy uncertainty, values are entered as fuzzy numbers. NAIADE defines four basic types of fuzzy numbers:

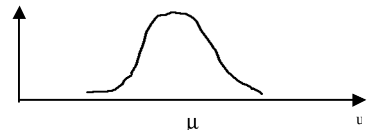
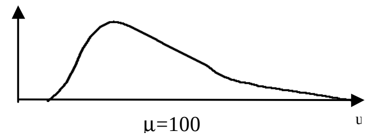
| **Membership function** | **Parameters to Determine** | **Shape** |
|---|---|---|
| *Gaussian*<br>$\mu_G(x)=\begin{cases}e^{-k(x-m)^2} \text{ for } l\le x\le u\\ 0 \text{ otherwise}\end{cases}$ | l, m, u, k<br>with k≥0 |  |
| *Flat*<br>$\mu_F(x)=\begin{cases}0 \text{ for } x\le l\\ 2((x-l)/(m1-l))^2 \text{ for } l<x\le b1\\ 1-2((m1-x)/(m1-l))^2 \text{ for } b1\le x<m1\\ 1 \text{ for } m1\le x\le m2\\ 1-2((x-m2)/(u-m2))^2 \text{ for } m2<x\le b2\\ 2((x-l)/(m1-l))^2 \text{ for } b2<x\le u\\ 0 \text{ for } x\ge u\end{cases}$ | l, m1, m2, u<br><br>b1=(l+m1)/2<br>b2=(m2+u)/2 |  |

| | | |
|---|---|---|
| *Left Right General*<br><br>$\mu_{LRG}(x)=\begin{cases} 0 \text{ for } x\leq l \\ 2((x-l)/(m-l))^2 \text{ for } l<x\leq b1 \\ 1-2((m-x)/(m-l))^2 \text{ for } b1\leq x<m \\ 1 \text{ for } x=m \\ 1-2((x-m)/(u-m))^2 \text{ for } m<x\leq b2 \\ 2((x-l)/(m-l))^2 \text{ for } b2<x\leq u \\ 0 \text{ for } x\geq u \end{cases}$ | l, m, u<br><br>b1=(l+m)/2<br>b2=(m+u)/2 |  |
| *Symmetrical*<br><br>$\mu_S(x)=\begin{cases} 0 \text{ for } x\leq l \\ 2((x-l)/(m-l))^2 \text{ for } l<x\leq b1 \\ 1-2((x-m)/(m-l))^2 \text{ for } b1\leq x<b2 \\ 2((x-m)/(u-m))^2 \text{ for } b2<x\leq u \\ 0 \text{ for } x\geq u \end{cases}$ | l, u<br><br>m=(l+u)/2<br>b1=(l+m)/2<br>b2=(m+u)/2 |  |

The task for the Decision Maker is to define which type of membership function best suites the kind of information available. NAIADE next uses the corresponding membership function in order to calculate overall preferences.

**Quantitative stochastic criteria.** When dealing with criteria of stochastic nature, NAIADE defines four types of density functions:

| **Probability density functions** | **Parameters to determine** | **Shape** |
|---|---|---|
| *Uniform*<br><br>$f_U(x)=\begin{cases} 1/(b-a) \text{ for } x\in[b, a] \\ 0 \text{ otherwise} \end{cases}$ | a, b<br><br>$\mu=(a+b)/2$<br>$\sigma=((a^2+b^2+ab)/3)$^0.5 |  |
| *Triangular*<br><br>$f_T(x)=\begin{cases} (b-\lvert x-a\rvert)/b^2 \text{ for } (a-b)\leq x\leq(a+b) \\ 0 \text{ otherwise} \end{cases}$ | a, b<br><br>$\mu=a$<br>$\sigma=b/6$^0.5 |  |

| | | |
|---|---|---|
| *Normal*<br><br>$f_N(x)=(1/(2\pi\sigma)^{0.5})e^{(-(x-\mu)^\wedge 2)/(2\sigma^\wedge 2)}$ | $\mu$, $\sigma$ |  |
| *LogNormal*<br><br>$f_{LN}(x)=(1/((2\pi\sigma)^{0.5}x))e^{(-(\log(x)-\chi)^\wedge 2)/(2\phi^\wedge 2)}$ | $\chi$, $\phi$ with $\phi>0$<br><br>$\mu=e(\chi+(\phi^2)/2)$<br>$\sigma=(e^{\phi^\wedge 2}(e^{\phi^\wedge 2}-1)\ e^{2\chi})^{0.5}$ |  |

The task for the Decision Maker is to establish parameters of the best suited density function.

**Quantitative numeric and fuzzy:** Idem quantitative fuzzy criteria

Considering the nature of criteria, Maria made the following inputs in NAIADE:

NAIADE IMPUT TABLE

| **CRITERION** | **Type** | **Black** | **Green** | **Grey** |
|---|---|---|---|---|
| Distance covered | Quantitative crisp | 95144Km. | 50480Km. | 26286Km. |
| Asked price | Quantitative fuzzy | approx. $10500 LeftRight | approx. $13500 LeftRight | approx. $16000 LeftRight |
| Taxes | Quantitative crisp | $100 | $150 | $150 |
| Fuel consumption | Quantitative fuzzy | approx. 0.0606 $/Km. Gaussian | approx. 0.0758 $/Km Gaussian | approx. 0.0758 $/Km. Gaussian |
| Cost of repair | Quantitative fuzzy | approx. $690 LeftRight | approx. $260 LeftRight | approx. $85 LeftRight |
| Cost of spare-parts | Qualitative | M | B | B |
| Technical condition | Qualitative | MLG | G | G |
| Body | Qualitative | MLG | G | G |
| Painting | Qualitative | MLG | G | G |
| Color | Qualitative | VG | G | MLB |
| Noise | Qualitative | MLG | G | G |
| General look | Qualitative | VG | G | MLG |
| Ergonomics | Qualitative | MLG | MLG | MLG |
| Stereo | Qualitative | G | MLG | MLG |
| Air conditioning | Qualitative | B | G | B |
| Seat cover | Qualitative | G | G | G |
| Roof cover | Qualitative | G | G | G |

| | | | | |
|---|---|---|---|---|
| Command panel | Qualitative | MLG | G | G |
| Risk of swindle | Qualitative | G | G | G |
| Legal obligations | Qualitative | G | G | G |
| Fuel type | Qualitative | P | G | G |
| Recyclable materials | Qualitative | M | M | M |
| Attractive? | Qualitative | MLB | M | M |
| Alarm | Qualitative | G | B | B |
| Lock | Qualitative | G | B | B |

## Step 4: Computing

Given two alternatives (*a*,*b*), the preference of *a* over *b* is expressed by means of the following six *preference relations*:

| **Preference relation** | **Functions** |
|---|---|
| *Much better* | $\mu_{>>}(d)=\begin{cases} 0, \text{ for } d<0 \\ 1/(1+((C^2_{>>}(\sqrt{2}-1))/d^2)^2), \text{ for } d\geq 0 \end{cases}$ |
| *Better* | $\mu_{>>}(d)=\begin{cases} 0, \text{ for } d<0 \\ 1/(1+(C^2_{>}/d^2)), \text{ for } d\geq 0 \end{cases}$ |
| *Approximately equal* | $\mu_{\cong}(d) = e^{-((\log 2/C_{\cong})\lvert d \rvert)}, \forall d$ |
| *Equal* | $\mu_{==}(d) = e^{-((\log 2/(C_{=} * C_{=})(d*d))}, \forall d$ |
| *Worse* | $\mu_{<}(d)= \begin{cases} 0, \text{ for } d<0 \\ 1/(1+(C^2_{<}/d^2)), \text{ for } d\geq 0 \end{cases}$ |
| *Much worse* | $\mu_{<<}(d)=\begin{cases} 0, \text{ for } d<0 \\ 1/(1+((C^2_{<<}(\sqrt{2}-1))/d^2)^2), \text{ for } d\geq 0 \end{cases}$ |
| The X axis represents the semantic distance *d* between alternatives and the Y axis is the credibility index level. | |

As explained earlier, the shape of preference relations is defined by means of establishing *crossover values* ($C_{==}$, $C_{\cong}$, $C_{>}$, $C_{>>}$) which are points where the functions equal 0.5. In the case of qualitative data, crossover values are established by setting the *credibility levels* of preference relations *much better* ($\mu_{>>}$), *better* ($\mu_{>}$), *approximately equal* ($\mu_{\cong}$), *equal* ($\mu_{==}$), *worse* ($\mu_{<}$) and *much worse* ($\mu_{<<}$), for the semantic distance between linguistic variables "*moderate*" and "*good*". In the case of quantitative data, crossover values are defined directly.

This means that (as it has been shown is Step 3) the Decision Maker quantitatively evaluates alternatives by choosing the best linguistic variable for the performance of every alternative on a given subjective criterion. Based on those linguistic variables, the model calculates the *semantic distance* of every pair of alternatives as follows:

Given two fuzzy sets $\mu_{A1}(x)$ y $\mu_{A2}(x)$, let's define:

$$f(x)=k_1\mu_{A1}(x) \text{ and } g(y)=k_2\mu_{A2}(y)$$

where $f(x)$ and $g(y)$ are two functions obtained by scaling the ordinates of $\mu_{A1}(x)$ and $\mu_{A2}(x)$ through $k_1$ and $k_2$, such that:

$$\int_{-\infty,+\infty} f(x)dx=\int_{-\infty,+\infty} g(y)dy=1$$

The semantic distance $S_d(f(x),g(x))$ between fuzzy sets $f(x)$ y $g(x)$ is defined as follows:

If $f(x)$: $X=[x_L,x_U]$ and $g(y)$: $Y=[x_{L'},x_{U'}]$, (where sets X and Y can be non-bounded), then,

$$\mathbf{S_d(f(x),g(x))=\int_X\int_Y|x-y|f(x)g(y)dxdy.}$$

Based on the just calculated semantic distance of every pair of alternatives, *preference intensity indexes* $\mu_*(a,b)_m$ are determined from the *preference relation functions*.

This allows to calculate an aggregated *preference intensity index* $\mu_*(a,b)$, which expresses the intensity of preference of one alternative over another, taking in to account preferences on each criterion $\mu_*(a,b)_m$:

$$\mu_*(a,b)=(\Sigma_{m=1,..,M}\max(\mu_*(a,b)_m-\alpha\ , 0)/(\Sigma_{m=1,..,M}|\mu_*(a,b)_m-\alpha|)$$

where:

- $_*$ stands for >>, >, ≅, ==, << and <
- M is the number of criteria
- $\alpha$ is a parameter used to express minimum requirements of credibility indexes. Only those criteria whose indexes are above the $\alpha$ threshold will be counted positively in the aggregation.
- $\mu_*(a,b)_m$ is the preference intensity index of criterion "m"
- $0\leq\mu_*(a,b)\leq 1$

- $\mu_*(a,b)=0$ if any $\mu_*(a,b)_m$ is more than $\alpha$
- $\mu_*(a,b)=1$ if $\mu_*(a,b)_m \geq \alpha \; \forall m$ and $\mu_*(a,b)_m > \alpha$ for at least one criterion

The next step is to calculate the *entropy* $H_*(a,b)$ of every preference intensity index. Entropy is an index varying from 0 to 1 used to measure the *variance* of credibility indexes that are above the $\alpha$ threshold and around the point of maximum fuzziness (i.e. the crossover value). An entropy of 0 means that there is no hesitation (either definitely credible or definitely non-credible), and an entropy of 1 means that the criteria is contradictory.

Preference intensity indexes $\mu_*(a,b)$ and correspondent entropies $H_*(a,b)$ are utilized to calculate the *degrees of truth* of the following statements:

" According to most of the criteria:
- ✓ *a* is *better* than *b*"
- ✓ the Decision Maker is *indifferent* between *a* and *b*"
- ✓ *a* is *worse* than *b*"

The preference relations *better*, *indifferent* and *worse* are calculated as follows:

$$\omega_{better}(a,b)=(\mu_{>>}(a,b) \wedge C_{>>}(a,b) + \mu_{>}(a,b) \wedge C_{>}(a,b))/(C_{>>}(a,b) + C_{>}(a,b))$$
$$\omega_{indifferent}(a,b)=(\mu_{=}(a,b) \wedge C_{=}(a,b) + \mu_{\cong}(a,b) \wedge C_{\cong}(a,b))/(C_{=}(a,b) + C_{\cong}(a,b))$$
$$\omega_{worst}(a,b)=(\mu_{<<}(a,b) \wedge C_{<<}(a,b) + \mu_{<}(a,b) \wedge C_{<}(a,b))/(C_{<<}(a,b) + C_{<}(a,b))$$

where $C_*(a,b) = 1 - H^*(a,b)$ is the associated entropy level over the preference intensity index, and the ^ operator can be replaced by the minimum operator or the Zimmermann-Zysno operator, according to the type of compensation required.

Next, the "*according to most of the criteria*" $\tau(\omega)$ operator is implemented as follows:

$$\tau(\omega)=\begin{cases} 1 & \forall\ \omega \geq 0.8 \\ 3.33\omega-1.66 & \forall\ 0.5 \leq \omega \leq 0.8 \\ 0 & \forall\ \omega \leq 0.5 \end{cases}$$

which filters the values of $\omega_{better}$, $\omega_{indifferent}$ and $\omega_{worst}$.

For NAIADE to calculate final ranking of alternatives, two partial rankings have to be calculated first:

$$\phi^+(a)=(\Sigma_{n=1..(N-1)}(\mu_{>>}(a,n) \wedge C_{>>}(a,n)+\mu_{>}(a,n) \wedge C_{>}(a,n))/(\Sigma_{n=1..(N-1)}C_{>>}(a,n)+\Sigma_{n=1..(N-1)}C_{>}(a,n))$$

$$\phi^-(a)=(\Sigma_{n=1..(N-1)}(\mu_{<<}(a,n) \wedge C_{<<}(a,n)+\mu_{<}(a,n) \wedge C_{<}(a,n))/(\Sigma_{n=1..(N-1)}C_{<<}(a,n)+\Sigma_{n=1..(N-1)}C_{<}(a,n))$$

where: → $\phi^+(a)$ is a partial preference intensity index based on the *better* and *much better* preference relations and, with a value ranging from 0 to 1, indicates how much *better a* is with respect to the remaining alternatives.

→ $\phi^-(a)$ is a partial preference intensity index based on the *worse* and *much worse* preference relations and, with a value ranging from 0 to 1, indicates how much *worse a* is with respect to all other alternatives.
→ N is the number of alternatives.
→ ^ is a generic operator that can be replaced by the minimum operator and the Zimmermann-Zysno operator, according to the type of compensation required.

The final ranking comes from the intersection of $\phi^+(a)$ and $\phi^-(a)$.

## Step 5: Defining parameters for calculation

For NAIADE to perform calculations, the following parameters must be defined by the Decision Maker:

⇨ Number of iterations in semantic distance: more iterations means more precision but slower calculation.
⇨ Number of iterations in integral calculation: the expected value of the number of the semantic distance.
⇨ Parameter for Minkowski distance: used only in equity analysis
⇨ Minimum requirement for fuzzy relation (α):
⇨ Operator

- Simple product
- Minimum operator: no compensation
- Zymmerman-Zysno operator: allows varying degrees of compensation according to (γ) Degree of compensation (γ) : from 0 minimum compensation to 1 maximum compensation

As recommended by the software authors, Maria employed the following default values:

| **Parameter** | **Default Value** |
|---|---|
| Number of iterations in semantic distance | 100 |
| Number of iterations in integral calculations | 100 |
| Parameter for Minkowski distance | 2 |
| Minimum requirement for fuzzy relations (α) | 0.4 |
| Operator | Minimum operator |

## Step 6: Getting results from NAIADE

NAIADE yields the following results:

| $\phi^+$ | $\phi^-$ | **Intersection** |
|---|---|---|
| 0.23 Black | 0.12 Green | Black → Grey |
| ↓ | ↓ | Green → Grey |
| 0.14 Green | 0.14 Black | |
| ↓ | ↓ | |
| 0.09 Grey | 0.19 Grey | |

## Comments on NAIADE

NAIADE constitutes, within the models analyzed in this paper, the most complete in terms of types of uncertainty that can be taken into account. Nevertheless, the software can be improved in a few aspects:

- ⇨ When dealing with quantitative fuzzy criteria, linguistic variables are not applicable, but fuzzy numbers. Moreover, too deep understanding of all parameters is required in order to use NAIADE properly.
- ⇨ To understand NAIADE procedure, the user must have knowledge of preference modeling theory, calculus and fuzzy sets theory, which is rarely the case for practitioners. This may cause difficulties to justify the use of NAIADE in front of persons with not very deep mathematical skills.
- ⇨ It can be difficult to interpret the meaning of crossover values. For this reason, it is recommended to use default values given by NAIADE.
- ⇨ There can be confusion when evaluating qualitative criteria that can be minimized. For instance risk: High risk must be defined as “bad”? Is “perfect” the maximum score and “extremely bad” the minimum score?
- ⇨ In NAIADE there is always the same semantic distance between *Very good and Good* Vs. *Moderate and Bad*. This gives arbitrariness to the model and may be not always true.
- ⇨ NAIADE concedes the same weight to all criteria, which may not be true in many real world applications.

## ***Comparing information requirements of Decision Making models***

| **Information requirement** | **Electre** | **MacBeth** | **Promethee** | **AHP** | **NAIADE** |
|---|---|---|---|---|---|
| Direct definition of weights of importance of every criterion | ✓ | | ✓ | | |
| Use of measurement scales defined by default | | ✓ | | ✓ | ✓ |
| Use of measurement scales defined by the Decision Maker | ✓ | | ✓ | | |
| Definition of thresholds | ✓ | | ✓ | | |
| Direct evaluation of alternatives with respect to each criterion | ✓ | | ✓ | | ✓ |
| Pairwise comparisons of attractiveness | | | | ✓ | |
| Pairwise comparisons of difference of attractiveness | | ✓ | | | |
| Definition of impact descriptors | | ✓ | | | |
| Definition of impact levels (levels "good" and "bad") | | ✓ | | | |
| Use of preference relations | | | ✓ | | ✓ |
| Use of hierarchies | | ✓ | | ✓ | |
| Use of membership functions | | | ✓ | | ✓ |

## ***Conclusions***

1) Different types of uncertainty can be taken into account when using Multi Criteria Decision Aiding Methods. This fact is of great importance given that the presence of incomplete information is not the exception in modern Socio-Economic systems and market structures that are highly complex. In order to work out realistic models of problems, various factors, both quantitative and qualitative, must be taken into account in decision making. Hardly any problem can be faced in the light of a single criterion without compromising the ability of suggesting wise solutions. Even in what has been traditionally considered purely monetary environments, such as enterprises and corporations, human and social factors must be considered if aiming at genuine progress. Employees and customers perceptions are critical success factors that ought to be considered by managers, just as social and political perceptions are critical success factors for governments. The use of qualitative criteria in decision processes with social connotations is even more obvious. Social conflicts, forecasts and environmental aspects can be better accounted through the use of preference modeling concepts, because of a simple reason: it does not require monetarization of intangibles. In Multi Criteria Decision Making procedures, there is no need to turn the outcomes into monetary dimensions.

2) In case of subjective criteria, some models ask the Decision Maker to qualify a given alternative using a predefined verbal scale like "*Perfect, Very good, Good, Moderate, Bad, Very Bad, Extremely Bad*". Certainly, when the Decision Maker uses one of these verbal

statements, s/he is in fact comparing the element with a prototype in his/her mind. This can be a source of ambiguity, specially when several individuals take part on the process. It seems to be preferable to use pairwise comparisons instead of direct evaluations.

3) Setting weights of relative importance is a task to be performed carefully. In some models, such as AHP, ELECTRE and Promethee, comparison among criteria is performed as follows:

- Which one is the most important: Price or Comfort?
- The answer: Price
- How much more important is Price than Comfort?
- The answer: Price is *very strongly more important* than Comfort

Although this answer seems to be very natural, it actually means that Price is, say, seven times more important than Comfort. This can be interpreted in at least two different ways:

a. A decrease in one unit of price is equivalent, in terms of satisfaction, to an increase of seven units of comfort, given that that the units of comfort and price are properly set (i.e. $1or $1000?).

b. An increase from the worst level of comfort to the best level of comfort is seven times more satisfying than an increase from the worst level of price to the best level of price. For this to have sense, it is necessary to set the limits meaning “worst” and “best” of both criteria.

The process of transforming ordinal values into cardinal ones, i.e. establishing values for ordinal verbal statements, can have a dose of arbitrariness. Establishing a value of *seven* for the verbal statement “*very strongly more important*” may be arbitrary. This can became a problem if the ordinal values are treated as cardinal for computation purposes. For this reason, a heavier usage of fuzzy numbers may be advisable for development of comprehensive Decision Support Systems.

4) The role of the analyst is of foremost importance in a good Multi Criteria Decision Aiding process. The process of determination of criteria, for example, is a central and difficult task. Normally, a multidisciplinary approach is required, in which expert advise in different relevant fields must be exercised. This can be even more difficult when more than one stakeholder intervene in the process. Although modeling of preferences through mathematical concepts may be a “sine qua non” activity, the process of problem structuring is the most challenging and important activity within a decision making process. Establishing criteria, creating alternatives, gearing people to achieve agreements, creating consensus, establishing priorities and relative importances, as well as obtaining sincere and bias-free judgments from Decision Maker’s are all tasks led by the analyst, who in general does not have deep knowledge on the topic relevant to the decision making process being performed. For this reason, MSc. and BSc. research is suggested on problem structuring processes, organizational behavior and human cognitive and decision processes.

5) Based on the implementation of five Decision Support Systems, we present a review of what is thought to be desirable characteristics of a Decision Support System:

A Decision Support System may:

- allow consideration of all types of uncertainty (see Annex A).
- be mathematically rigorous.
- be user friendly (i.e. easy understanding of inputs and outputs).
- allow graphical representation of information.
- facilitate a comprehensive approach of the whole Decision Making Process, starting from the structuring phase.
- allow to perform evaluations through pairwise comparisons.
- allow to express relative importance of criteria
- avoid direct weighting of criteria.
- check inconsistency of judgments.

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## ***ANNEX : A brief discussion about uncertainty***

To our view, uncertainty can be understood as the lack of ability:

⇨ to accurately know or predict something
⇨ to be sure or confident about something
⇨ to precisely determine or decide about something

We can define six sources of uncertainty:

➡ *Uncertainty from imprecision*: I am uncertain because my instrument is not precise enough.
➡ *Uncertainty from ambiguity*: I am uncertain because it depends on the context.
➡ *Uncertainty from presumption*: I am uncertain because I can not see the future, but:
- it can be modeled with an experiment. [Repetition, stochastic]
- there are similar experiences in the past. [Extrapolation, heuristic]
- I have enough experience to foresee the outcome. [Experience, heuristic]

➡ *Uncertainty from subjectivity*: I am uncertain because there are no means of measuring it.
➡ *Uncertainty from dynamism*: I am uncertain because it varies with time.
➡ *Uncertainty from contradiction*: I am uncertain because the information I got is contradictory.

There are many aspects that can be identified as uncertain in this specific case study. The price, for example, is composed by quantities that can be precisely known such as the asked price (in the case bargaining is not allowed) and the amount of taxes to be paid. On the contrary, it is not possible or too expensive to know the precise amount of gas consumption or the cost of repairing the vehicle to the best possible condition. Be noted that the cost of repair given by the expert in the technical diagnosis is a subjective approximation of the real cost of repair. The subjective approximation is based 1) on the cost of the spare-parts that the technician believes should be replaced after his relatively quick examination of the element and 2) the amount of time s/he believes could be spent in the repairing process. Both, the price of spare-parts to be used on the repairing process and the labor cost for repairing the vehicle are uncertain, despite the fact that the cost of the spare-parts and the labor cost are very well known. Uncertainty comes from the fact that the technician can not know the real state of the vehicle until s/he completely opens the element to be repaired. This kind of uncertainty is of the type "*I am uncertain because I can not see the future, but I have enough experience to foresee the outcome*" and the forecast about the real state of the vehicle will came in the form of a subjective statement.

Criteria like Aesthetics and Comfort offer a special type of uncertainty that we wish to comment with interest. The uncertainty inherent to these criteria is of the type "*I am uncertain because there are no means of measuring it*". Notice that "*no means of*" means that there is no instrument capable to quantitatively measure that criterion or that it is possibly too expensive or unjustifiably difficult to do it. The noise while driving, for example, can be measured using a device that measures the degree of noise in decibels, but it could be too expensive, too difficult to find such a device or simply too time consuming. Given the importance of the criterion "noise while driving", it is easier to just get in the car, drive trough a rough street and qualify the amount of noise with a sentence like "*it is not so much…*" or "*what a mess!…*". Subjective criterion like *aesthetics* offers a challenge to the analyst. It can be difficult to put a number that describes how much one likes the car. Probably it would be much easier to directly compare two vehicles and to say which one is the best in terms of aesthetics.